Interacting Conformal Particles and Tachyonic Carroll Strings

Joaquim Gomis

Universitat Barcelona

Based on several papers

R. Casalbuoni, D. Dominici, JG
2306.02614, 2403.02152

A Non-Lorentz Primer
E. Bergshoeff, J. Figueroa O'Farrill
2206.12177

Carroll workshop
17 April
ESI, Vienna
2024

(Electric) Carroll particle 1405.2264

Carroll limit, Bergshoeff, Longhi, JG

Non-linear Realizations

coadjoint orbit method Duval, Gibbons, Horvathy 1402.0657

$$L = p_\mu \dot{x}^\mu - \frac{e}{2}(p^2 + m^2)$$

Carroll limit

ω dimensionless, $\omega \to \infty$

$$\begin{cases} x^0 = \dfrac{t}{\omega}, \quad p_0 = -E\omega \\ m = M\omega \\ e = -\dfrac{\tilde{e}}{\omega^2} \end{cases}$$

$$L = -E\dot{t} + \vec{p}\,\dot{\vec{x}} - \frac{\tilde{e}}{2}(E^2 - M^2)$$

Electric Carroll does not move.

Symmetries.

$$\begin{cases} \delta t = \vec{\beta}\,\vec{x} \\ \delta \vec{x} = 0 \end{cases} \quad \begin{cases} \delta E = 0 \\ \delta \vec{p} = \vec{\beta} E \end{cases}$$

Enlargement of Carroll.

Generator of point transformation

$$G = -E\,\xi^0(\vec{x},t) + p_i\,\xi^i(\vec{x},t) + \gamma(\vec{x},t,e)\,\pi_e$$

$\dot{G} = 0$ on the surface of primary constraints

$$M \neq 0 \quad \begin{cases} \delta t = \xi^0(\vec{x}) \\ \delta x^i = \xi^i(\vec{x}) \end{cases} \quad \delta \tilde{e} = 0$$

$$M = 0 \quad \begin{cases} \delta t = \xi^0(t, \vec{x}) \\ \delta x^i = \xi^i(\vec{x}) \\ \delta \tilde{e} = 2\tilde{e}\, \partial_t \xi^0(t, \vec{x}) \end{cases}$$

$$\delta E = \{E, G\}$$

$\downarrow$

These infinite dimensional symmetries contain

1) Conformal Carroll transformations

The generators are

$$H = E, \quad \vec{P} = \vec{p}, \quad \vec{G} = E\vec{x}, \quad \vec{J} = \vec{x} \times \vec{p}$$

$$D = -Et + \vec{p}\vec{x}, \quad K^0 = -E\vec{x}^2, \quad \vec{K} = 2D\vec{x} - x^2 p$$

Bagchi et al 1609.06203

2) Conformal Carroll $\simeq$ BMS

Duval et al
1402.5894

$$X = Y^A(x)\frac{\partial}{\partial x^A} + \left[\frac{\lambda}{N}u + T(x)\right]\frac{\partial}{\partial u}$$

<u>Wave equations</u>

$$(\hat{E}^2 - m^2)|\psi\rangle = 0 \to -\left(\frac{d^2}{dt^2} + m^2\right)\phi(\vec{x},t) = 0$$

Lagrangian

$$\mathcal{L} = \frac{1}{2}\phi\left[-\partial_t^2 - m^2\right]\phi.$$

when $m = 0$ Henneaux - Salgado-Rebolledo
2109.06708

Magnetic Carroll particle

We use the mapping among Galilei and Carroll

$$\underset{\text{gal}}{H} \longleftrightarrow \underset{\text{carroll}}{\vec{P}}$$

Carroll Transf.

$$\vec{p} \longrightarrow E \to 0 \quad \text{double nilpotency}$$

$$\delta(\vec{p}^{\,2} - m^2) = 2\vec{\beta} E \vec{p} = 2(\vec{\beta}\vec{p})E$$

mass-shell constraint
of mass Galilei
particle

Souriau (1970)

New Carroll mass constraint

$$\vec{p}^{\,2} - m^2 - XE = 0\,, \quad \delta X = 2\vec{p}\vec{\beta}$$

$$L^e_{mag} = -E\dot{t} + \vec{p}\dot{\vec{x}} - \frac{e}{2}\left(\vec{p}^{\,2} - m^2 - XE\right)$$

$$= -E\dot{t} + \vec{p}\dot{\vec{x}} - \frac{e}{2}\left(\vec{p}^{\,2} - m^2\right) - \mu E$$

Tachyonic Carroll particle

Vandoren et
2110.02319
Kleinschmidt
JG
2202.05206

$$L = m\sqrt{\dot{x}^2} \xrightarrow[x^0 = \frac{t}{c}]{} m\sqrt{\dot{\vec{x}}^2}$$

$l_m = M$

Constraints $\vec{P}^2 - m^2 = 0$, $E = 0$

The associated canonical action coincides with the magnetic Carroll particle

Quantization a la Dirac — Figueroa O'Farrill, Perez, Prohazka 2307.05674

$$\begin{cases} (\vec{\nabla}^2 + m^2)\,\phi(t,\vec{x}) = 0 \\ \partial_t \phi(t,\vec{x}) = 0 \end{cases}$$

A possible associated Lagrangian is

$$\mathcal{L} = +\frac{1}{2}\phi(\vec{\nabla}^2 + m^2)\phi - X\partial_t\phi$$

Review paper

$$\begin{cases} \delta_C \phi = \vec{\beta}\cdot\vec{x}\,\partial_t\phi \\ \delta_C X = \vec{\beta}\cdot\vec{x}\,\partial_t X - \beta^i\partial_i\phi \end{cases}$$

Transport Term

equations of motion

$$\begin{cases} \vec{\nabla}^2\phi + m^2\phi + \dot{X} = 0 \\ \dot{\phi} = 0 \end{cases}$$

$X = $ const irreducible

$X \neq$ const indecomposable

Classification all coadjoint orbits of Carroll — Figueroa-O'Farrill et al 2305.06730 boost transformation

$$\delta\begin{pmatrix}\phi\\ \chi\end{pmatrix} = \begin{pmatrix}\vec{\beta}\vec{x}\partial_t\,, & 0\\ -\beta^i\partial_i\,, & \vec{\beta}\vec{x}\partial_t\end{pmatrix}\begin{pmatrix}\phi\\ \chi\end{pmatrix}$$

indescomposable representation

Conformal Carroll Transformations of X

$$\delta X = -e\,\vec{p}\,\vec{B} \quad \text{Carroll boost}$$

$$\delta X = \epsilon_D X$$

$$\delta X = 2\left[e_a(b^0\vec{p}\cdot\vec{x} - t\,\vec{p}\cdot\vec{b}) + X\,\vec{b}\vec{x}\right]$$

Non-equivalence of Carroll limits

Casalbuoni,
Dominici, J.G
2403.02152

Consider the relativistic lagrangian with einbein, which is equivalent to the canonical one, and its Carroll limit

$$L = \frac{1}{2}\frac{\dot{x}^2}{e} - \frac{1}{2}m^2 e \longrightarrow L = \frac{1}{2}\frac{\dot{\vec{x}}^2}{e} - \frac{1}{2}m^2 e$$

$$\begin{cases} x^0 = \dfrac{t}{\omega} \\ m = m \\ e = e \end{cases}$$

The constraints are

$\pi_e = 0$, $E = 0$ primary

$\vec{p}^{\,2} + m^2 = 0$ secondary

The model is inconsistent unless $m = 0$

When $m = 0$

$$L = -E\dot{t} + \vec{p}\,\dot{\vec{x}} - \frac{e}{2}\vec{p}^{\,2} - \mu E$$

irregular constraint

$$N_{dof} = \frac{1}{2}\left[2(D+2) - 2\times 4\right] = D-2$$

Following Miskovic - Zanelli
hep-th/0302033

The linearity constraint is $\vec{p}=0$.

The canonical action is

$$L := -E\dot{t} + \vec{p}\,\dot{\vec{x}} - \vec{\lambda}\,\vec{p} - \mu E$$ "vacuum"

$$N_{dof} = \frac{1}{2}\left[2(D+D-1+1) - 2(D-1+D-1+1+1)\right] = 0$$

No - local degrees of freedom, "Topological"

Two Interacting Conformal Carroll particles

Relativistic Conformal interacting particles

Casalbuoni, Dominici 1404.5766

Two free non-interacting conformal particles

$$S = -\int d\tau \left(\frac{\dot{x}_1^2}{2e_1} + \frac{\dot{x}_2^2}{2e_2} \right).$$

dilatation   inversion

$$x_\mu \to \lambda x_\mu, \quad e_a \to \lambda^2 e_a, \quad x^\mu \to \frac{x^\mu}{x^2}, \quad \dot{x}^2 \to \frac{\dot{x}^2}{x^4}$$

inversion → translation → inversion

$$\frac{x^\mu}{x^2} \to \underbrace{\frac{x^\mu}{x^2} + a^\mu}_{= y^\mu} \to \frac{1}{y^2} = \frac{x^\mu}{1 + 2(a x) + a^2 x^2}$$

$$\underbrace{r_\mu = x_{1\mu} - x_{2\mu}}$$

$$r^2 \to \frac{r^2}{x_1^2 x_2^2}$$

$$S = -\int d\tau \left(\frac{\dot{x}_1^2}{2e_1} + \frac{\dot{x}_2^2}{2e_2} + \frac{\alpha^2}{4} \frac{\sqrt{e_1 e_2}}{r^2} \right)$$

$$\underbrace{p_1^2 - \frac{\alpha^2}{4}\sqrt{\frac{e_2}{e_1}}\frac{1}{r^2} = 0}_{\phi_1} \qquad \underbrace{p_2^2 - \frac{\alpha^2}{4}\sqrt{\frac{e_1}{e_2}} = 0}_{\phi_2}$$

$$\downarrow$$

$$p_1^2 p_2^2 - \frac{\alpha^4}{16 r^4} = 0$$

$\downarrow$ eliminating einbein

$$S = -\alpha \int d\tau \left(\frac{\dot{x}_1^2 \dot{x}_2^2}{r^4} \right)^{1/4}$$

Carroll limit

Casalbuoni, Dominici, J. G. 2306.02614

$$\begin{cases} p_a^0 = \omega E_a, \\ x_a^0 = \frac{1}{\omega} t_a \\ \tilde{e}_a = e_a \omega^2 \\ \tilde{\alpha}^2 = \frac{\alpha^2}{\omega^2} \end{cases}$$

$$L = -E_1 \dot{t}_1 + \vec{p}_1 \dot{\vec{x}}_1 - E_2 \dot{t}_2 + \vec{p}_2 \dot{\vec{x}}_2$$

$$+ \frac{\tilde{e}_1}{2}\left(E_1^2 - \frac{\alpha^2}{4}\sqrt{\frac{\tilde{e}_2}{\tilde{e}_1}} \frac{1}{\vec{r}^2} \right)$$

electric type

$$+ \tilde{e} \left(E^2 - \tilde{\alpha}^2 \sqrt{\tilde{e}_1} \; 1 \; \right)$$

$\ldots c_2 E_2 \quad \overline{4} \sqrt{\overline{E_2} \; \overline{\vec r}^2}$

$$\underline{E_1 E_2 - \frac{\alpha^2}{4\vec r^{\,2}} = 0} \quad \text{constraint without einbeins}$$

$$\underline{\left[\partial_{t_1}\partial_{t_2} + \frac{\alpha^2}{4\vec r^{\,2}}\right] \Phi(t_1, t_2, \vec x_1, \vec x_2) = 0}$$

Carroll transformations

$$\delta_c x_a^i = \epsilon^i - \lambda^{ij} x_a^j, \quad \delta_c p_a^i = -\lambda^{ij} p_a^j + \beta^i E_a$$

$$\delta_c t_a = h + \vec\beta \cdot \vec x_a, \quad \delta_c E_a = 0, \quad \delta \ell_a = 0$$

Infinite dimensional symmetries

$$G = \sum_a \xi_a^0 (t_1, t_2, \vec x_1, \vec x_2) E_a -$$

$$- \vec\xi_a (t_1, t_2, \vec x_1, \vec x_2) \vec p_a +$$

$$+ \gamma_a (t_1, t_2, \vec x_1, \vec x_2) \ell_a$$

Killing equations

$$\frac{\partial \xi_a^j (t_1, t_2, \vec x_1, \vec x_2)}{\partial t_c} = 0, \quad a, c = 1, 2$$

$$\Rightarrow \quad \xi_a^j = \xi_a^j (x_1, x_2)$$

Define $\gamma_a = e_a \tilde{\gamma}_a$

$$\frac{\partial \xi^{0}(t_1, t_2, \vec{x}_1, \vec{x}_2)}{\partial t_c} = -\frac{1}{2}\delta_{ac}\tilde{\gamma}_a \,, \quad a,b,c = 1,2$$

and

$$\frac{\alpha^2}{r'^2}(\vec{\xi}_1 - \vec{\xi}_2)\cdot\vec{r} = \frac{\alpha^2}{2}\frac{\delta \vec{r}'^2}{r^2} = -\frac{\alpha^2}{2}(\tilde{\gamma}_1 + \tilde{\gamma}_2)$$

$a = 1,2$

$$\xi_a^{0} = \delta t_a = -\frac{1}{2}\tilde{\gamma}_a(\vec{x}_1, \vec{x}_2)\,t + h_a(x_1, x_2)$$

↑ linear.

$$G_2 \subset \mathcal{G}_1 \times \mathcal{G}_2$$

Two conformal Carroll particles:

A Tachyonic model

$$L = -E_1 \dot{t}_1 + \vec{p}_1 \dot{\vec{x}}_1 - E_2 \dot{t}_2 + \vec{p}_2 \dot{\vec{x}}_2$$

$$-\frac{e_1}{2}\left(\vec{p}_1^{\,2} - \frac{\alpha^2}{4}\sqrt{\frac{e_2}{e_1}}\frac{1}{r^2}\right) - \frac{e_2}{2}\left(\vec{p}_2^{\,2} - \frac{\alpha^2}{4}\sqrt{\frac{e_1}{e_2}}\frac{1}{r^2}\right)$$

$$-X_1 E_1 - X_2 E_2$$

This suggest that a galilean invariant model is given

$$L_G = -E\dot{t}_1 + \vec{p}_1\dot{\vec{x}}_1 - E_2\dot{t}_2 + \vec{p}_2\dot{\vec{x}}_2$$

$$-\frac{e_1}{2}\left(\vec{p}_1^{\,2} - \frac{\alpha^2}{4}\sqrt{\frac{e_2}{e_1}}\frac{1}{r^{12}}\right) - \frac{e_2}{2}\left(\vec{p}_2^{\,2} - \frac{\alpha^2}{4}\sqrt{\frac{e_1}{e_2}}\frac{1}{r^{12}}\right)$$

Carroll Tachyonic string

An Electric Carroll string

Cardona, Pons, JG 1605.05483

$$L = p\dot{x} - \frac{e}{2}\left(p^2 + T^2 x'^2\right) - \mu p x'$$

$$\downarrow \qquad x^0 = \frac{t}{w},\ p^0 = wE$$
$$\tilde{e} = \frac{e}{w},\ \tilde{T} = wT$$

$$L_c = -E\dot{t} + \vec{p}\dot{\vec{x}} - \frac{e}{2}\left(-E^2 + T^2 x'^2\right) - \mu\left(Et' + \vec{p}\vec{x}'\right)$$

In the case of an string limit

$$L = -E\dot{t} + \vec{p}\dot{\vec{x}} - \frac{e}{2}\left(h_{\mu\nu}p^\mu p^\nu + T^2 x_\perp'^2\right)$$
$$-\mu\left(Et' + \vec{p}\vec{x}'\right)$$

Carroll string symmetries

$$\begin{cases} \delta x^\mu = \omega^\mu{}_\nu x^\nu + \omega^\mu{}_i x^i + \varepsilon^\mu \\ \delta P_\mu = \omega_\mu{}^\nu P_\nu \end{cases} \qquad \begin{cases} \delta x^i = \omega^i{}_j x^j + \varepsilon^i \\ \delta P_i = \omega_i{}^\mu P_\mu + \omega_i{}^j P_j \end{cases}$$

There are infinite global symmetries

Tachyonic string

Casalbuoni, Dominici, J.G. 2306.02614

$$\mathcal{L} = \frac{1}{2e}\dot{x}^2 - \frac{\mu}{e}\dot{x}x' + \frac{1}{2}\frac{\mu^2}{e}x'^2 + eT^2 x'^2$$

$$\Downarrow$$

$$\mathcal{L} = \frac{1}{2e}\dot{\vec{x}}^2 - \frac{\mu}{e}\dot{\vec{x}}\vec{x}' + \frac{1}{2}\frac{\mu^2}{e}\vec{x}'^2 + \frac{T^2}{2}e\vec{x}'^2$$

$E = 0$ primary

$$\mathcal{H} = \vec{p}^2 - T^2\vec{x}'^2$$
$$\mathcal{Z} = \vec{p}\cdot\vec{x}'$$

secondary constraints

Canonical action ($t = 0$, $E = 0$)

$$\mathcal{L} = \dot{\vec{x}}\vec{p} - \frac{1}{2}\left(\vec{p}^2 - T^2\vec{x}'^2\right) - \mu\vec{p}\vec{x}'$$