

Carrollian Amplitudes, Celestial Symmetries and Twistor Space

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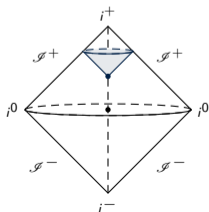
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Lionel Mason and Akshay Yelleshpur Srikant



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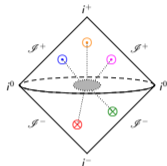
How to formulate flat space holography?

- Correspondence between **gravity in asymptotically flat spacetimes** and a **lower-dimensional field theory** without gravity.
- **Bottom-up** approaches to build candidates for **holographic duals**.
- Two proposals for flat space holography in 4d:
 - ⇒ **Celestial holography**: the dual theory is a **2d CFT** living on the **celestial sphere** S^2 .
 [de Boer-Solodukhin '03] [He-Mitra-Strominger '15] [Kapec-Mitra-Raclariu-Strominger '16] [Cheung-de la Fuente-Sundrum '16] [Pasterski-Shao-Strominger '17] [Pasterski-Shao '17] [Donnay-Puhm-Strominger '18] [Stieberger-Taylor '18] [Pate-Raclariu-Strominger-Yuan '19] [Adamo-Mason-Sharma '21] ...
 - ⇒ **Carrollian holography**: the dual theory is a **3d Carrollian CFT** living at **null infinity** $\mathcal{I} \simeq \mathbb{R} \times S^2$.
 [Arcioni-Dappiaggi '03] [Dappiaggi-Moretti-Pinamonti '06] [Barnich-Compère '07] [Bagchi '10] [Barnich '12] [Bagchi-Detournay-Fareghbal-Simon '12] [Barnich-Gomberoff-Gonzalez '12] [Bagchi-Basu-Grumiller-Riegler '15] [Ciambelli-Marteau-Petkou-Petropoulos-Siampos '18] [Donnay-Fiorucci-Herfray-Ruzziconi '22] ...
- The two proposals are **related** [Donnay-Fiorucci-Herfray-Ruzziconi '22] [Bagchi-Banerjee-Basu-Dutta '22].
- In this talk: **focus on the Carrollian approach**.

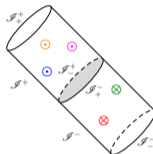


Carrollian holography

Gravity in $(d+1)$ -dimensional asymptotically flat spacetime



d -dimensional conformal Carrollian field theory at \mathcal{I}



Carrollian holography

- Asymptotic symmetries of the bulk theory = global symmetries in the dual theory.
 \implies Asymptotic symmetries in flat space = BMS symmetries [Bondi-van der Burg-Metzner '62] [Sachs '62]
- Isomorphism: $\mathfrak{bms}_{d+1} \simeq \mathcal{CCarr}_d$. [Duval-Gibbons-Horvathy '14] and $\text{Poincaré}_{d+1} \simeq \text{Global } \mathcal{CCarr}_d$
- The dual theory is a d -dimensional Carrollian CFT (= theory exhibiting conformal Carroll/BMS spacetime symmetries)
 \implies Can be constructed by taking $c \rightarrow 0$ of standard relativistic CFTs, see e.g.
 [Schild '77] [Isberg-Lindstrom-Sundborg-Theodoridis '94] [Barnich-Gomberoff-Gonzalez '12] [Duval-Gibbons-Horvathy '14] [Bagchi-Mehra-Nandi '19] ...
- Carrollian holography follows a similar pattern than AdS/CFT correspondence: $(d + 1)$ -dimensional bulk / d -dimensional boundary duality.
 \implies Naturally arises from a flat limit procedure ($\ell \rightarrow \infty$).
 \implies The flat limit in the bulk induces a Carrollian limit ($c \rightarrow 0$) at the boundary.
 [Bagchi '10] [Barnich-Gomberoff-Gonzalez '12] [Ciambelli-Marteanu-Petkou-Petropoulos-Siampos '18] [Compère-Fiorucci-Ruzziconi '19]
 [Campoleoni-Delfante-Pekar-Petropoulos-Rivera Betancour-Vilatte '23]

An overview of previous works...

- Flat limit successfully exploited in 3d gravity:
 - ① Gravitational solution space and symmetries ($\text{Witt} \oplus \text{Witt} \implies \mathfrak{bms}_3$) [Barnich-Gomberoff-Gonzalez '12] [Bagchi-Fareghbal '12]
 - ② Entropy matching between flat space cosmologies and Carrollian CFT [Barnich '12] [Bagchi-Detournay-Fareghbal-Simon '13]
 - ③ Entanglement entropy formulae [Li-Takayanagi '11] [Bagchi-Basu-Grumiller-Riegler '14] [Jiang-Song-Wen '17]
 - ④ Holographic computation of boundary Carrollian stress tensor correlators [Detournay-Grumiller-Scholler-Simon '14] [Bagchi-Grumiller-Merbis '15] [Hartong '16]
 - ⑤ Effective Carrollian CFT action at \mathcal{I} [Barnich-Gomberoff-Gonzalez '13]
 - ⑥ Holographic anomaly in flat space [Campoleoni-Ciambelli-Delfante-Marteau-Petropoulos-Ruzziconi '22]

...
- Flat limit of the gravitational phase space and symmetries also works in 4d. [See Sabrina's talk]
 [Poole-Skenderis-Taylor '18] [Compère-Fiorucci-Ruzziconi '19] [Compère-Fiorucci-Ruzziconi '20] [Geiller-Zwikel '22] [Ciambelli-Pasterski-Tabor '24]
- Flat limit in the fluid/gravity correspondence. [Ciambelli-Marteau-Petkou-Petropoulos-Siampos '18] [Freidel-Jai-akson '22]
 [Campoleoni-Delfante-Pekar-Petropoulos-Rivera Betancour-Vilatte '23]
- Carrollian CFT actions at the boundary of 4d flat space. [Adamo-Casali-Skinner '14] [Barnich-Nguyen-Ruzziconi '22]
- **Important missing piece:** relation with 4d \mathcal{S} -matrix and celestial amplitudes? [See Stephan's talk]
 [Donnay-Fiorucci-Herfray-Ruzziconi '22] [Bagchi-Banerjee-Basu-Dutta '22] [Saha '23] [Mason-Ruzziconi-Yellespur Srikant '23] [Liu-Long-Ye '24]
 [Have-Nguyen-Prohazka-Salzer '24] [Stieberger-Taylor-Zhu '24]
- **Objective** of this talk: Define Carrollian amplitudes [Mason-Ruzziconi-Yellespur Srikant '23].
 \implies Encode the \mathcal{S} -matrix into boundary Carrollian CFT correlators.

Bondi coordinates

- Planar Bondi coordinates $\{u, r, z, \bar{z}\}$ ($u, r \in \mathbb{R}, z \in \mathbb{C}$):

$$X^\mu = u \partial_z \partial_{\bar{z}} q^\mu(z, \bar{z}) + r q^\mu(z, \bar{z}), \quad q^\mu(z, \bar{z}) \equiv \frac{1}{\sqrt{2}} (1 + z\bar{z}, z + \bar{z}, -i(z - \bar{z}), 1 - z\bar{z}).$$

- Minkowski metric:

$$ds^2 = -2du dr + 2r^2 dz d\bar{z}.$$

- Induced Carrollian structure at future/past null infinity $\mathcal{I}^\pm = \{r \rightarrow \pm\infty\}$:

$$ds_{\mathcal{I}}^2 = q_{ab} dx^a dx^b = 0 du^2 + 2 dz d\bar{z}, \quad n^a \partial_a = \partial_u$$

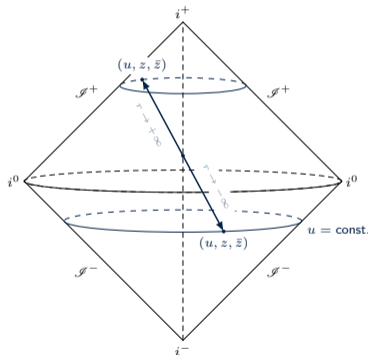
with $x^a = (u, z, \bar{z})$ the boundary coordinates. [Penrose '63] [Geroch '77] [Ashtekar '14]

- Natural (antipodal) identification between \mathcal{I}^+ and \mathcal{I}^- .
- BMS/conformal Carroll symmetries: [Bondi-van der Burg-Metzner '62] [Sachs '62]

$$\xi^a \partial_a = \left[\mathcal{T} + \frac{u}{2} (\partial \mathcal{Y} + \bar{\partial} \bar{\mathcal{Y}}) \right] \partial_u + \mathcal{Y} \partial + \bar{\mathcal{Y}} \bar{\partial}$$

with defining property: $\mathcal{L}_\xi q_{ab} = 2\alpha q_{ab}$, $\mathcal{L}_\xi n^a = -\alpha n^a$, $\alpha = \frac{1}{2} (\partial \mathcal{Y} + \bar{\partial} \bar{\mathcal{Y}})$.

- $\mathcal{T} = \mathcal{T}(z, \bar{z})$ is the supertranslation parameter;
- $\mathcal{Y} = \mathcal{Y}(z)$, $\bar{\mathcal{Y}} = \bar{\mathcal{Y}}(\bar{z})$ are the superrotation parameters satisfying the conformal Killing equation. [Barnich-Troessaert '10]



Massless scattering in flat space

Can we encode the bulk \mathcal{S} -matrix into boundary Carrollian CFT correlators?

- Strategy: start from the bulk operators, and deduce the boundary operators at \mathcal{I} . [Ashtekar '81] [Arcioni-Dappiaggi '03] [Strominger '17] [Donnay-Fiorucci-Herfray-Ruzziconi '22]

- Consider a spin- s ($s = 0, 1, 2, \dots$) massless field in flat space:

$$\phi_l^{(s)}(X) = \frac{K_{(s)}}{16\pi^3} \sum_{\alpha=\pm} \int \omega d\omega d^2w \left[a_{\alpha}^{(s)}(\omega, w, \bar{w}) \varepsilon_l^{*\alpha}(w, \bar{w}) e^{i\omega q^\mu X_\mu} + a_{\alpha}^{(s)}(\omega, w, \bar{w})^\dagger \varepsilon_l^{\alpha}(w, \bar{w}) e^{-i\omega q^\mu X_\mu} \right]$$

with $l = (\mu_1 \mu_2 \dots \mu_s)$ and

$$p^\mu(\omega, w, \bar{w}) = \omega q^\mu(w, \bar{w}), \quad q^\mu(w, \bar{w}) = \frac{1}{\sqrt{2}} \left(1 + w\bar{w}, w + \bar{w}, -i(w - \bar{w}), 1 - w\bar{w} \right),$$

$$\varepsilon_{\mu_1 \dots \mu_s}^{\pm}(\bar{q}) = \varepsilon_{\mu_1}^{\pm}(\bar{q}) \varepsilon_{\mu_2}^{\pm}(\bar{q}) \dots \varepsilon_{\mu_s}^{\pm}(\bar{q}), \quad \varepsilon_{\mu}^{+}(\bar{q}) = \partial_w q_\mu = \frac{1}{\sqrt{2}} (-\bar{w}, 1, -i, -\bar{w}), \quad \varepsilon_{\mu}^{-}(\bar{q}) = [\varepsilon_{\mu}^{+}(\bar{q})]^*.$$

- Taking $r \rightarrow \infty$ (stationary phase approximation), we find the boundary values:

$$\bar{\phi}_{z \dots z}^{(s)}(u, z, \bar{z})^{\text{out}} = \lim_{r \rightarrow +\infty} \left(r^{1-s} \phi_{z \dots z}^{(s)}(u, r, z, \bar{z}) \right) = -\frac{iK_{(s)}}{8\pi^2} \int_0^{+\infty} d\omega \left[a_+^{(s)\text{out}}(\omega, z, \bar{z}) e^{-i\omega u} - a_-^{(s)\text{out}}(\omega, z, \bar{z})^\dagger e^{i\omega u} \right] \quad \text{at } \mathcal{I}^+,$$

$$\bar{\phi}_{z \dots z}^{(s)}(u, z, \bar{z})^{\text{in}} = \lim_{r \rightarrow -\infty} \left(r^{1-s} \phi_{z \dots z}^{(s)}(u, r, z, \bar{z}) \right) = -\frac{iK_{(s)}}{8\pi^2} \int_0^{+\infty} d\omega \left[a_+^{(s)\text{in}}(\omega, z, \bar{z}) e^{-i\omega u} - a_-^{(s)\text{in}}(\omega, z, \bar{z})^\dagger e^{i\omega u} \right] \quad \text{at } \mathcal{I}^-.$$

\implies Insertion operators for a massless scattering between \mathcal{I}^- (in) and \mathcal{I}^+ (out).

Carrollian primaries

- Conformal Carrollian primary field $\Phi_{(k, \bar{k})}(u, z, \bar{z})$:

$$\delta_{\xi} \Phi_{(k, \bar{k})} = \left[\left(\mathcal{T} + \frac{u}{2} (\partial \mathcal{Y} + \bar{\partial} \bar{\mathcal{Y}}) \right) \partial_u + \mathcal{Y} \partial + \bar{\mathcal{Y}} \bar{\partial} + k \partial \mathcal{Y} + \bar{k} \bar{\partial} \bar{\mathcal{Y}} \right] \Phi_{(k, \bar{k})}, \quad (k, \bar{k}): \text{Carrollian weights.}$$

(analogue of primary field in CFT)

\implies The Carrollian primaries transform in unitary representations of global $\mathcal{CCatt}_3 \simeq \text{Poincaré}_4$ [Nguyen-West '23] [Nguyen '23].

- Carrollian correlators living at \mathcal{S} : $\langle \Phi_{(k_1, \bar{k}_1)}(u_1, z_1, \bar{z}_1) \Phi_{(k_2, \bar{k}_2)}(u_2, z_2, \bar{z}_2) \dots \rangle$.
- Remark: if $\Phi_{(k, \bar{k})}(u, z, \bar{z})$ is a Carrollian primary, then $\partial_u \Phi_{(k, \bar{k})}(u, z, \bar{z})$ is also a Carrollian primary with shifted weights $(k + \frac{1}{2}, \bar{k} + \frac{1}{2})$.

Carrollian holography identification

- Carrollian operators = boundary value of bulk operators:

[Arcioni-Dappiaggi '03] [Dappiaggi-Moretti-Pinamonti '05] [Donnay-Fiorucci-Herfray-Ruzziconi '22]

$$\Phi_{(k, \bar{k})}^{\epsilon=+1}(u, z, \bar{z}) = \bar{\phi}_{z \dots z}^{(s)\text{out}}(u, z, \bar{z}), \quad \Phi_{(k, \bar{k})}^{\epsilon=-1}(u, z, \bar{z}) = \bar{\phi}_{z \dots z}^{(s)\text{in}}(u, z, \bar{z})^\dagger$$

\implies This implies $k = \frac{1+\epsilon J}{2}$, $\bar{k} = \frac{1-\epsilon J}{2}$ with $\epsilon = \pm 1$ for out/in.

\implies For gravity ($s = 2$), $\Phi_{(k, \bar{k})}^{\epsilon=+1}(u, z, \bar{z}) \equiv C_{zz}(u, z, \bar{z})$ ($J = 2$ and $(k, \bar{k}) = (\frac{3}{2}, -\frac{1}{2})$).

- Carrollian correlators = scattering amplitudes in position space at \mathcal{I} :

$$\begin{aligned} \langle \Phi_{(k_1, \bar{k}_1)}^{\epsilon_1}(u_1, z_1, \bar{z}_1) \dots \Phi_{(k_n, \bar{k}_n)}^{\epsilon_n}(u_n, z_n, \bar{z}_n) \rangle &= \prod_{i=1}^n \left(\int_0^{+\infty} \frac{d\omega_i}{2\pi} e^{i\epsilon_i \omega_i u_i} \right) \mathcal{A}_n \left(\{\omega_1, z_1, \bar{z}_1\}_{J_1}^{\epsilon_1}, \dots, \{\omega_n, z_n, \bar{z}_n\}_{J_n}^{\epsilon_n} \right) \\ &= \mathcal{C}_n \left(\{u_1, z_1, \bar{z}_1\}_{J_1}^{\epsilon_1}, \dots, \{u_n, z_n, \bar{z}_n\}_{J_n}^{\epsilon_n} \right) \end{aligned}$$

\implies Amplitudes in position space at \mathcal{I} = Carrollian amplitudes. [Donnay-Fiorucci-Herfray-Ruzziconi '22] [Mason-Ruzziconi-Yellespur Srikant '23]

- Extrapolate dictionary for Carrollian holography:

$$\langle \Phi_{(k_1, \bar{k}_1)}^{\epsilon_1}(u_1, z_1, \bar{z}_1) \dots \Phi_{(k_n, \bar{k}_n)}^{\epsilon_n}(u_n, z_n, \bar{z}_n) \rangle = \lim_{r \rightarrow \epsilon_\infty} \langle r^{1-s_1} \phi^{(s_1)}(u_1, r_1, z_1, \bar{z}_1) \dots r^{1-s_n} \phi^{(s_n)}(u_n, r_n, z_n, \bar{z}_n) \rangle$$

Carrollian Ward identities

- Consistent with the (global) conformal Carrollian Ward identities:

$$\sum_{i=0}^n \left[\left(\mathcal{T}(z_i, \bar{z}_i) + \frac{u_i}{2} (\partial_{z_i} \mathcal{Y}(z_i) + \partial_{\bar{z}_i} \bar{\mathcal{Y}}(\bar{z}_i)) \right) \partial_{u_i} + \mathcal{Y}(z_i) \partial_{z_i} + \bar{\mathcal{Y}}(\bar{z}_i) \partial_{\bar{z}_i} + k_i \partial_{z_i} \mathcal{Y}(z_i) + \bar{k}_i \partial_{\bar{z}_i} \bar{\mathcal{Y}}(\bar{z}_i) \right] \langle \Phi_{(k_1, \bar{k}_1)}(u_1, z_1, \bar{z}_1) \dots \Phi_{(k_n, \bar{k}_n)}(u_n, z_n, \bar{z}_n) \rangle = 0$$

where

$$\mathcal{T}(z, \bar{z}) = 1, z, \bar{z}, z\bar{z}, \quad \mathcal{Y}(z) = 1, z, z^2, \quad \bar{\mathcal{Y}}(\bar{z}) = 1, \bar{z}, \bar{z}^2$$

⇒ The low-point correlation functions are completely fixed by the conformal Carrollian symmetries.

⇒ The higher-point functions provide non-trivial information on the dynamics.

- In particular, for the 2-point function [Chen-Liu-Zheng, '21]:

$$\langle \Phi_{(k_1, \bar{k}_1)}(u_1, z_1, \bar{z}_1) \Phi_{(k_2, \bar{k}_2)}(u_2, z_2, \bar{z}_2) \rangle = \begin{cases} \frac{\alpha}{(u_1 - u_2)^{k_1 + k_2 + \bar{k}_1 + \bar{k}_2 - 2}} \delta^{(2)}(z_1 - z_2) \delta_{k_1 + k_2, \bar{k}_1 + \bar{k}_2} & \text{(Electric branch)} \\ \frac{\beta}{(z_1 - z_2)^{k_1 + k_2} (\bar{z}_1 - \bar{z}_2)^{\bar{k}_1 + \bar{k}_2}} \delta_{k_1, k_2} \delta_{\bar{k}_1, \bar{k}_2} & \text{(Magnetic branch)} \end{cases}$$

⇒ Electric branch relevant for massless scattering.

[Donnay-Fiorucci-Herfray-Ruzziconi '22] [Bagchi-Banerjee-Basu-Dutta '22] [Mason-Ruzziconi-Yellespur Srikant '23]

⇒ The $\delta^{(2)}(z_1 - z_2)$ distribution is a standard feature of Carrollian CFT (light cones shrink into lines), not undesirable.

Modified Mellin transform

- If $\Phi_{(k, \bar{k})}(u, z, \bar{z})$ is a Carrollian primary, then $\partial_u \Phi_{(k, \bar{k})}(u, z, \bar{z})$ is also a Carrollian primary with shifted weights $(k + \frac{1}{2}, \bar{k} + \frac{1}{2})$.
- Remark: the leading order of the Peeling is $\partial_u^{|J|} \Phi$. For gravity, $\Psi_4^0 = \partial_u^2 C_{zz} \implies$ Transforms as primary under the full $\mathcal{CC}\alpha\tau\tau_4$ algebra. [Mason-Ruzziconi-Yellespur Srikant '23]
- Descendants:

$$\begin{aligned} \mathcal{C}_n^{m_1 \dots m_n} \left(\{u_1, z_1, \bar{z}_1\}_{J_1}^{\epsilon_1}, \dots, \{u_n, z_n, \bar{z}_n\}_{J_n}^{\epsilon_n} \right) &= \partial_{u_1}^{m_1} \dots \partial_{u_n}^{m_n} C_n \left(\{u_1, z_1, \bar{z}_1\}_{J_1}^{\epsilon_1}, \dots, \{u_n, z_n, \bar{z}_n\}_{J_n}^{\epsilon_n} \right) \\ &= \prod_{i=1}^n \left(\int_0^{+\infty} \frac{d\omega_i}{2\pi} (i\epsilon_i \omega_i)^{m_i} e^{i\epsilon_i \omega_i u_i} \right) \mathcal{A}_n \left(\{\omega_1, z_1, \bar{z}_1\}_{J_1}^{\epsilon_1}, \dots, \{\omega_n, z_n, \bar{z}_n\}_{J_n}^{\epsilon_n} \right) \\ &= \langle \partial_{u_1}^{m_1} \Phi_{(k_1, \bar{k}_1)}^{\epsilon_1}(u_1, z_1, \bar{z}_1) \dots \partial_{u_n}^{m_n} \Phi_{(k_n, \bar{k}_n)}^{\epsilon_n}(u_n, z_n, \bar{z}_n) \rangle \end{aligned}$$

- In particular, $\tilde{\mathcal{C}}_n \equiv \mathcal{C}_n^{1 \dots 1}$.
- Redundant to encode the \mathcal{S} -matrix, but can be useful to get rid of IR divergences (see example of the 2-point function).
- Remark: analytic continuation $m_i = \delta_i - 1$ ($\delta_i \in \mathbb{C}$) gives

$$\prod_{i=1}^n \left(\int_0^{+\infty} \frac{d\omega_i}{2\pi} (i\epsilon_i \omega_i)^{\delta_i - 1} e^{i\epsilon_i \omega_i u_i} \right) \mathcal{A}_n \left(\{\omega_1, z_1, \bar{z}_1\}_{J_1}^{\epsilon_1}, \dots, \{\omega_n, z_n, \bar{z}_n\}_{J_n}^{\epsilon_n} \right),$$

\implies Modified Mellin transform used to regularize Mellin transform of graviton amplitudes. [Banerjee '18] [Banerjee-Ghosh-Pandey-Saha '20]

\implies Clarifies the link with the alternative approach to Carrollian amplitudes. [Bagchi-Banerjee-Basu-Dutta '22]

Relation between Carrollian CFT and celestial CFT

- Celestial amplitudes obtained by Mellin transform [de Boer-Solodukhin '03] [Pasterski-Shao-Strominger '17] [Pasterski-Shao '17]:

$$\mathcal{M}_n \left(\{ \Delta_1, z_1, \bar{z}_1 \}_{J_1}^{\epsilon_1}, \dots, \{ \Delta_n, z_n, \bar{z}_n \}_{J_n}^{\epsilon_n} \right) = \prod_{i=1}^n \left(\int_0^{+\infty} d\omega_i \omega_i^{\Delta_i - 1} \right) \mathcal{A}_n \left(\{ \omega_1, z_1, \bar{z}_1 \}_{J_1}^{\epsilon_1}, \dots, \{ \omega_n, z_n, \bar{z}_n \}_{J_n}^{\epsilon_n} \right)$$

$$\equiv \langle \mathcal{O}_{\Delta_1, J_1}^{\epsilon_1}(z_1, \bar{z}_1) \dots \mathcal{O}_{\Delta_n, J_n}^{\epsilon_n}(z_n, \bar{z}_n) \rangle$$

- Relation between Carrollian and celestial amplitudes [Donnay-Fiorucci-Herfray-Ruzziconi '22]:

$$\mathcal{M}_n \left(\{ \Delta_1, z_1, \bar{z}_1 \}_{J_1}^{\epsilon_1}, \dots, \{ \Delta_n, z_n, \bar{z}_n \}_{J_n}^{\epsilon_n} \right) = \prod_{i=1}^n \left((-i\epsilon_i)^{\Delta_i} \Gamma[\Delta_i] \int_{-\infty}^{+\infty} \frac{du_i}{(u_i - i\epsilon_i \varepsilon)^{\Delta_i}} \right) \mathcal{C}_n \left(\{ u_1, z_1, \bar{z}_1 \}_{J_1}^{\epsilon_1}, \dots, \{ u_n, z_n, \bar{z}_n \}_{J_n}^{\epsilon_n} \right)$$

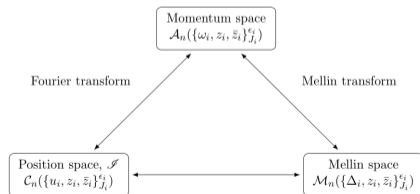
- Relation between Carrollian and celestial operators:

$$\mathcal{O}_{\Delta_i, J_i}^{\epsilon_i}(z_i, \bar{z}_i) = (-i\epsilon_i)^{\Delta_i} \Gamma[\Delta_i] \int_{-\infty}^{+\infty} \frac{du_i}{(u_i - i\epsilon_i \varepsilon)^{\Delta_i}} \Phi_{(k_i, \bar{k}_i)}^{\epsilon_i}(u_i, z_i, \bar{z}_i)$$

⇒ Exchange between time and conformal dimension.

⇒ Three scattering bases (ω, u, Δ) [Donnay-Pasterski-Puhm '22] [Freidel-Pranzetti-Raclariu '22].

⇒ Extrapolate dictionary in celestial holography [Pasterski-Puhm-Trevisani '21].



Two-point Carrollian amplitude

- Two-point amplitude (one incoming and one outgoing particle):

$$\mathcal{A}_2(\{\omega_1, z_1, \bar{z}_1\}_{J_1}^-, \{\omega_2, z_2, \bar{z}_2\}_{J_2}^+) = \kappa_{J_1, J_2}^2 \pi \frac{\delta(\omega_1 - \omega_2)}{\omega_1} \delta^{(2)}(z_1 - z_2) \delta_{J_1, J_2},$$

- Carrollian two-point amplitude obtained by Fourier transform [Liu-Long '22] [Donnay-Fiorucci-Herfray-Ruzziconi '22]:

$$\begin{aligned} \mathcal{C}_2(\{u_1, z_1, \bar{z}_1\}_{J_1}^-, \{u_2, z_2, \bar{z}_2\}_{J_2}^+) &= \frac{1}{4\pi^2} \int_0^{+\infty} d\omega_1 \int_0^{+\infty} d\omega_2 e^{-i\omega_1 u_1} e^{i\omega_2 u_2} \mathcal{A}_2(\{\omega_1, z_1, \bar{z}_1\}_{J_1}^-, \{\omega_2, z_2, \bar{z}_2\}_{J_2}^+) \\ &= \frac{\kappa_{J_1, J_2}^2}{4\pi} \int_0^{+\infty} \frac{d\omega}{\omega} e^{-i\omega(u_1 - u_2)} \delta^{(2)}(z_1 - z_2) \delta_{J_1, J_2}. \end{aligned}$$

- The divergent integral in the last line can be regularized. Instead, let us consider the correlator of ∂_u -descendants:

$$\tilde{\mathcal{C}}_2(\{u_1, z_1, \bar{z}_1\}_{J_1}^-, \{u_2, z_2, \bar{z}_2\}_{J_2}^+) = \lim_{\varepsilon \rightarrow 0^+} \frac{\kappa_{J_1, J_2}^2}{4\pi} \frac{1}{(u_{12} - i\varepsilon)^2} \delta^{(2)}(z_{12}) \delta_{J_1, J_2}$$

\implies Standard solution of the Carrollian Ward identities (electric branch) for operators with fixed Carrollian weights $k + \bar{k} = 2$.
 [See Claudio's talk]

- Apply the Carroll/celestial relation:

$$\mathcal{M}_2(\{\Delta_1, z_1, \bar{z}_1\}_{J_1}^-, \{\Delta_2, z_2, \bar{z}_2\}_{J_2}^+) = 2\pi^2 \kappa_{J_1, J_2}^2 \delta(\Delta_1 + \Delta_2 - 2) \delta^{(2)}(z_{12}) \delta_{J_1, J_2},$$

[Pasterski-Shao-Strominger '17]

\implies Price to pay to go from 3d Carrollian CFT to 2d CFT: distributional low-point functions, not standard in CFT.

Three-point Carrollian amplitude

- The three-point amplitude generically vanishes in Lorentzian signature \implies Go to split (2, 2) signature.

$$p_i^\mu = \epsilon_i q_i^\mu = \epsilon_i \omega_i (1 + z_i \bar{z}_i, z_i + \bar{z}_i, z_i - \bar{z}_i, 1 - z_i \bar{z}_i).$$

Here (z_i, \bar{z}_i) are coordinates on a Poincaré patch of \mathcal{LT}_2 and $\epsilon_i = \pm 1$ labels the Poincaré patches.

- Using spinor-helicity notations, $p_{\alpha\dot{\alpha}} \equiv \sigma_{\alpha\dot{\alpha}}^\mu p_\mu = \kappa_\alpha \tilde{\kappa}_{\dot{\alpha}}$ and $[ij] = \tilde{\kappa}_{i\dot{\alpha}} \tilde{\kappa}_j^{\dot{\alpha}}$, at tree-level, the three-point amplitude reads as

$$\mathcal{A}_3(1^{J_1}, 2^{J_2}, 3^{J_3}) = \kappa_{J_1, J_2, J_3} [12]^{J_1+J_2-J_3} [23]^{J_2+J_3-J_1} [31]^{J_3+J_1-J_2} \delta^{(4)}(p_1 + p_2 + p_3), \text{ if } J_1 + J_2 + J_3 > 0$$

A similar expression exists for $J_1 + J_2 + J_3 + 2 < 0$.

- Three-point Carrollian amplitude (still determined by Ward identities):

[Banerjee-Ghosh-Pandey-Saha '20] [Salzer '23] [Mason-Ruzziiconi-Yellespur Srikant '23]

$$\begin{aligned} \tilde{\mathcal{C}}_3 = & \kappa_{J_1, J_2, J_3} \frac{-i\epsilon_1\epsilon_2\epsilon_3 \delta(z_{12}) \delta(z_{23})}{4(2\pi)^3} \Theta\left(-\frac{\bar{z}_{13}}{\bar{z}_{23}} \epsilon_1\epsilon_2\right) \Theta\left(\frac{\bar{z}_{12}}{\bar{z}_{23}} \epsilon_1\epsilon_3\right) |\bar{z}_{12}|^{J_1+J_2} |\bar{z}_{23}|^{J_2+J_3} |\bar{z}_{31}|^{J_3+J_1} \\ & \times (\text{sign } \bar{z}_{12})^{J_1+J_2-J_3+1} (\text{sign } \bar{z}_{23})^{J_2+J_3-J_1+1} (\text{sign } \bar{z}_{13})^{J_1+J_3-J_2+1} \frac{(i\epsilon_1 \text{sign}(\bar{z}_{23}))^{J_1+J_2+J_3+2} \Gamma(J_1 + J_2 + J_3 + 2)}{(\bar{z}_{23}u_1 - \bar{z}_{13}u_2 + \bar{z}_{12}u_3 + i\epsilon_1 \text{sign}(\bar{z}_{23})\epsilon)^{J_1+J_2+J_3+2}} \end{aligned}$$

for $J_1 + J_2 + J_3 + 2 > 0$ (similarly for $J_1 + J_2 + J_3 + 2 < 0$).

- Using Carroll/celestial correspondence ($\bar{h}_k = \frac{\Delta_k - J_k}{2}$):

$$\begin{aligned} \mathcal{M}_3 = & \frac{(-i)^{J_1+J_2+J_3} \pi}{2} \kappa_{J_1, J_2, J_3} \delta(z_{12}) \delta(z_{23}) \Theta\left(-\frac{\bar{z}_{13}}{\bar{z}_{23}} \epsilon_1\epsilon_2\right) \Theta\left(\frac{\bar{z}_{12}}{\bar{z}_{23}} \epsilon_1\epsilon_3\right) \frac{1}{\bar{z}_{12}^{\bar{h}_1+\bar{h}_2-\bar{h}_3} \bar{z}_{23}^{\bar{h}_2+\bar{h}_3-\bar{h}_1} \bar{z}_{13}^{\bar{h}_3+\bar{h}_1-\bar{h}_2}} \\ & \times (\text{sign } \bar{z}_{12})^{J_1+J_2-J_3} (\text{sign } \bar{z}_{23})^{J_2+J_3-J_1} (\text{sign } \bar{z}_{13})^{J_1+J_3-J_2} (\epsilon_1)^{\Delta_1} (\epsilon_2)^{\Delta_2} (\epsilon_3)^{\Delta_3} \delta(\Delta_1 + \Delta_2 + \Delta_3 + J_1 + J_2 + J_3 - 4). \end{aligned}$$

[Pasterski-Shao-Strominger '17]

Four-point Carrollian amplitude

- At tree-level the 4-point gluon MHV amplitude is given by

$$\mathcal{A}_4(1^{+1}, 2^{-1}, 3^{-1}, 4^{+1}) = \kappa_{1,1,-1}^2 \frac{\langle 23 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle} = \kappa_{1,1,-1}^2 \frac{\omega_2 \omega_3}{\omega_1 \omega_4} \frac{z_{23}^3}{z_{12} z_{34} z_{41}},$$

- Applying the Fourier transform yields the corresponding Carrollian amplitude (very similar computation for gravitons):
[Banerjee-Ghosh-Pandey-Saha '20] [Mason-Ruzziiconi-Yellespur Srikant '23]

$$\begin{aligned} \check{\mathcal{C}}_4(1^{+1}, 2^{-1}, 3^{-1}, 4^{+1}) &= \frac{\kappa_{1,1,-1}^2}{(2\pi)^4} \frac{z_{34}^2 \bar{z}_{14}^4 \bar{z}_{34}^2}{z^3 (1-z) z_{13}^3 z_{24} \bar{z}_{13}^5 \bar{z}_{24}^3} \delta(z - \bar{z}) \Theta\left(-z \left| \frac{z_{24}}{z_{12}} \right|^2 \epsilon_1 \epsilon_4\right) \Theta\left(\frac{1-z}{z} \left| \frac{z_{34}}{z_{23}} \right|^2 \epsilon_2 \epsilon_4\right) \\ &\quad \Theta\left(-\frac{1}{1-z} \left| \frac{z_{14}}{z_{13}} \right|^2 \epsilon_3 \epsilon_4\right) \times \frac{3!}{\left(u_4 - u_1 z \left| \frac{z_{24}}{z_{12}} \right|^2 + u_2 \frac{1-z}{z} \left| \frac{z_{34}}{z_{23}} \right|^2 - u_3 \frac{1}{1-z} \left| \frac{z_{14}}{z_{13}} \right|^2\right)^4}, \end{aligned}$$

- Using Carroll/celestial correspondence:

$$\begin{aligned} \mathcal{M}_4 &= \prod_{i=1}^4 \left(\int_{-\infty}^{+\infty} du_i (-i\epsilon_i)^{\Delta_i} \Gamma(\Delta_i - 1) u_i^{1-\Delta_i} \right) \check{\mathcal{C}}_4 \\ &= \prod_{i=1}^4 (-i\epsilon_i)^{\Delta_i} z^{-\frac{1}{3}} (1-z)^{\frac{5}{3}} \prod_{i<j} z_{ij}^{\frac{h}{3} - h_i - h_j} \bar{z}_{ij}^{\frac{\bar{h}}{3} - \bar{h}_i - \bar{h}_j} (-1)^{\Delta_2 + \Delta_4 + 1} 2\pi \delta(\Delta_1 + \Delta_2 + \Delta_3 + \Delta_4 - 4) \\ &\quad \delta(z - \bar{z}) \Theta\left(-z \left| \frac{z_{24}}{z_{12}} \right|^2 \epsilon_1 \epsilon_4\right) \Theta\left(\frac{1-z}{z} \left| \frac{z_{34}}{z_{23}} \right|^2 \epsilon_2 \epsilon_4\right) \Theta\left(-\frac{1}{1-z} \left| \frac{z_{14}}{z_{13}} \right|^2 \epsilon_3 \epsilon_4\right). \end{aligned}$$

[Pasterski-Shao-Strominger '17]

n -point MHV Carrollian amplitude

- Colour ordered MHV gluon amplitude (with $n + 1$ identified with 1):

$$\mathcal{A}_n(1^-, 2^-, 3^+, \dots, n^+) = \kappa_{1,1,-1}^{n-2} \frac{\langle 12 \rangle^4}{\prod_{j=1}^n \langle jj+1 \rangle} = \kappa_{1,1,-1}^{n-2} \frac{\omega_1 \omega_2}{\prod_{j=3}^n \omega_j} \frac{z_{12}^3}{\prod_{j=2}^n z_{jj+1}}$$

(similar formula for gravitons)

- Use the decomposition of the delta distribution [Schreiber-Volovich-Zlotnikov '17]:

$$\delta^{(4)}\left(\sum_{i=1}^n p_i\right) = \frac{1}{|\mathcal{U}_{1234}|} \prod_{l=1}^4 \delta(\omega_l - \omega_l^*), \quad \text{with} \quad \omega_l^* = -\frac{1}{\mathcal{U}_{1234}} \sum_{i=5}^n \omega_i \mathcal{U}_{li}$$

where

$$\mathcal{U}_{1234} = \det(q_1^\mu, \dots, q_4^\mu), \quad \mathcal{U}_{li} = \mathcal{U}_{1234}|_{l \rightarrow i}, \quad l = 1, 2, 3, 4; i = 5, \dots, n.$$

- Applying the Fourier transform [Mason-Ruzziiconi-Yellespur Srikant '23]:

$$\check{\mathcal{C}}_n(1^-, 2^-, 3^+, \dots, n^+) = \frac{\kappa_{1,1,-1}^{n-2}}{(2\pi)^n |\mathcal{U}_{1234}|} \frac{z_{12}^3}{\prod_{j=2}^n z_{jj+1}} \frac{\partial^4}{\partial u_1^2 \partial u_2^2} I_n$$

where the integral I_n can be computed explicitly:

$$I_n = \int \prod_{j=5}^n d\omega_j e^{i\omega_j L_j} \prod_{l=1}^4 \Theta(\omega_l^*) = (-1)^{n-4} \prod_{j=5}^n \frac{1}{L_j}$$

$$\text{with } L_j = \left(\epsilon_j u_j - \sum_{J=1}^4 \epsilon_J u_J \frac{\mathcal{U}_{Jj}}{\mathcal{U}_{1234}} \right)$$

- Non-trivial dynamical constraints on the dual Carrollian CFT.
- Similar expression for the MHV graviton amplitude (same integral I_n).
- Surprisingly simpler than its celestial counterparts involving Aomoto-Gelfand hypergeometric function. [Schreiber-Volovich-Zlotnikov '17]

Collinear limit and Carrollian OPE

- Collinear limit of two outgoing particles ($\epsilon_1 = \epsilon_2 = +1$):

$$\mathcal{A}_n \left(1^{J_1}, 2^{J_2}, 3^{J_3}, \dots, n^{J_n} \right) \xrightarrow{1||2} \sum_J \mathcal{A}_3 \left(1^{J_1}, 2^{J_2}, -P^{-J} \right) \frac{1}{\langle 12 \rangle [21]} \mathcal{A}_{n-1} \left(P^J, 3^{J_3}, \dots, n^{J_n} \right)$$

where J is the helicity of the exchanged particle.

- In the limit $z_{12} \rightarrow 0$, we obtain the Carrollian OPE block [Mason-Ruzziconi-Yellespur Srikant '23]:

$$\begin{aligned} & \Phi_{J_1}(u_1, z_1, \bar{z}_1) \Phi_{J_2}(u_2, z_2, \bar{z}_2) \\ & \sim -\frac{\kappa_{J_1, J_2, -J}}{2\pi} \frac{\bar{z}_{12}^p}{z_{12}} \int_0^1 dt t^{J_2 - J - 1} (1-t)^{J_1 - J - 1} \left(\frac{\partial}{\partial u} \right)^p \Phi_J(u, z_2, \bar{z}_2 + t\bar{z}_{12})|_{u=u_2 + tu_{12}} \\ & \sim -\frac{\kappa_{J_1, J_2, -J}}{2\pi z_{12}} \sum_{m, n=0}^{\infty} B(J_2 - J + m + n, J_1 - J) \frac{\bar{z}_{12}^{p+m} u_{12}^n}{m! n!} \left(\frac{\partial}{\partial \bar{z}_2} \right)^m \left(\frac{\partial}{\partial u_2} \right)^{p+n} \Phi_J(u_2, z_2, \bar{z}_2) \end{aligned}$$

with implicit sum on $p = J_1 + J_2 - J - 1$ with range determined by

$$p \geq 0, \quad |J_1 + J_2 - p - 1| \leq 2 \quad \text{and} \quad |J_1| \leq 2, \quad |J_2| \leq 2.$$

- Invariance under global $\mathcal{CCl}\mathfrak{rtt}_3$ explicitly checked \checkmark
- Using the Carroll/celestial correspondence, we recover the celestial OPE block

$$\mathcal{O}_{\Delta_1, J_1}(z_1, \bar{z}_1) \mathcal{O}_{\Delta_2, J_2}(z_2, \bar{z}_2) \sim -\kappa_{J_1, J_2, -J} \frac{\bar{z}_{12}^p}{z_{12}} \int_0^1 dt t^{2\bar{h}_1 + p - 1} (1-t)^{2\bar{h}_2 + p - 1} \mathcal{O}_{\Delta_1 + \Delta_2 + p - 1, J}$$

[Fan-Fotopoulos-Taylor '19] [Pate-Raclariu-Strominger-Yuan '19]

Celestial symmetries on Carrollian operators

- Celestial formulation of scattering amplitudes \implies Presence of the $w_{1+\infty}$ highlighted in the celestial OPE algebra. [Guevara-Himwich-Pate-Strominger '21] [Strominger '21]
- Can be traced back to the Penrose's non-linear graviton construction [Penrose '76] [Adamo-Mason-Sharma '21].
- Action of $w_{1+\infty}$ at $\mathcal{S}?$ \implies Can be deduced from Carrollian OPEs [Mason-Ruzziconi-Yelleshpur Srikant '23].
- Definite the (outgoing) soft operators [Guevara-Himwich-Pate-Strominger '21]

$$w^q(z, \bar{z}) = \lim_{\varepsilon \rightarrow 0} \frac{(-1)^{2q} \Gamma(2q)}{2\pi i \varepsilon} \int_{-\infty}^{\infty} \frac{d\bar{w}}{(\bar{z} - \bar{w})^{2q}} \mathcal{O}_{4-2q+\varepsilon, 2}(z, \bar{w}), \quad q = \frac{3}{2}, 2, \frac{5}{2}, \dots$$

- Decomposition in modes

$$w^q(z, \bar{z}) = \sum_{m=q-1}^{1-q} \frac{1}{\bar{z}^{m+1-q}} w_m^q, \quad q = \frac{3}{2}, 2, \frac{5}{2}, \dots$$

- $w_{1+\infty}$ algebra:

$$[w_m^p, w_n^q] = (m(q-1) - n(p-1)) w_{m+n}^{p+q-2}.$$

- Action of $w_{1+\infty}$ on Carrollian graviton operator [Mason-Ruzziconi-Yelleshpur Srikant '23]:

$$w^q(z_1, \bar{z}_1) \partial_{u_2} \Phi_2(u_2, z_2, \bar{z}_2) \sim -\frac{\kappa_{2,2,-2}}{z_{12}} \sum_{m=0}^{2q-3} (-1)^m (m+1) \bar{z}_{12}^{m+2-2q} (-iu_2)^{2q-3-m} (-i\partial_{u_2})^{2-m} \partial_{\bar{z}_2}^m \Phi_2(u_2, z_2, \bar{z}_2).$$

\implies Local for $q \leq \frac{5}{2}$ corresponding to universal soft theorems.

\implies Compatible with the $w_{1+\infty}$ action on celestial operators [Guevara-Himwich-Pate-Strominger '21] [Strominger '21]. \checkmark

\implies Compatible with the $w_{1+\infty}$ action on null data at \mathcal{S} derived from the radiative phase space [Freidel-Pranzetti-Raclaru '21] [Geiller '24]. \checkmark

From twistor space to Carrollian operators

- The action of $w_{1+\infty}$ on twistor space is local and has a geometric interpretation (Poisson diffeomorphisms). [Adamo-Mason-Sharma '21] [Bu-Heuveline-Skinner '22] [Mason '22]

- Two integral representations for zero rest mass null fields in flat space:

$$\partial^{\alpha\dot{\alpha}}\Psi_{\alpha_1\dots\alpha_n} = 0$$

- 1) Kirchoff-d'Adhémar formula:

$$\Psi_{\alpha_1\dots\alpha_n}(x) = \frac{1}{2\pi} \int_{\mathcal{S}} \lambda_{\alpha_1} \dots \lambda_{\alpha_n} \partial_u^{n+1} \Phi_J(x^{\alpha\dot{\alpha}} \lambda_{\alpha} \tilde{\lambda}_{\dot{\alpha}}, \lambda, \tilde{\lambda}) D\lambda D\tilde{\lambda}$$

where $|J| = 2n$, $D\lambda := \langle \lambda d\lambda \rangle$ and $D\tilde{\lambda} := [\tilde{\lambda} d\tilde{\lambda}]$, and we introduced homogeneous coordinates $(u, \lambda_{\alpha}, \tilde{\lambda}_{\dot{\alpha}}) = (\lambda_0 \tilde{\lambda}_{\dot{0}} u_B, \lambda_0 z_{\alpha}, \tilde{\lambda}_{\dot{0}} \bar{z}_{\dot{\alpha}})$.

- 2) Penrose transform:

$$\Psi_{\alpha_1\dots\alpha_n}(x) = \int_{S^1} \lambda_{\alpha_1} \dots \lambda_{\alpha_n} f(\lambda, x^{\alpha\dot{\alpha}} \lambda_{\alpha}) D\lambda$$

where $f(\lambda_{\alpha}, \mu^{\dot{\alpha}})$ is an homogeneous function on twistor space \mathbb{RP}^3 with homogenous coordinates $(\lambda_{\alpha}, \mu^{\dot{\alpha}})$.

- Natural to relate Carrollian CFT at \mathcal{S} and twistor space [Mason-Ruzziconi-Yellespur Srikant '23]:

$$\Phi_J(u, \lambda, \tilde{\lambda}) = \partial_u^{|J|} \int d^2\mu \delta(u - [\mu\tilde{\lambda}]) f(\lambda_{\alpha}, \mu^{\dot{\alpha}})$$

where we introduced homogeneous coordinates $(u, \lambda_{\alpha}, \tilde{\lambda}_{\dot{\alpha}}) = (\lambda_0 \tilde{\lambda}_{\dot{0}} u_B, \lambda_0 z_{\alpha}, \tilde{\lambda}_{\dot{0}} \bar{z}_{\dot{\alpha}})$.

- Example: gravity ($n = 4$, $|J| = 2$)

$$C_{ZZ}(u, \lambda, \tilde{\lambda}) = \partial_u^2 \int d^2\mu \delta(u - [\mu\tilde{\lambda}]) f(\lambda_{\alpha}, \mu^{\dot{\alpha}})$$

\implies Can map twistor space amplitudes to Carrollian amplitudes.

\implies Understand better why $w_{1+\infty}$ are symmetries at \mathcal{S} .

Summary and perspectives

- Two complementary approaches to flat space holography:



- The two approaches are related via integral transform ✓
- Carrollian holography is a useful path:
 - ⇒ Naturally related to AdS/CFT via $\ell \rightarrow \infty, c \rightarrow 0$ ✓
 - ⇒ Successful in 3d gravity ✓
 - ⇒ Consistent encoding of the S -matrix ✓
 - ⇒ Natural extrapolate dictionary. ✓
- Carrollian physics has applications beyond flat space holography:
Black hole, cosmology, condensed matter, fluid...
- Perspectives:
 - ⇒ Relations with AdS amplitudes? Carrollian limit of CFT correlators?
 - ⇒ Top-down models for Carrollian holography? via $c \rightarrow 0$ limit?

Thank you!

Carrollian algebra

- “Carroll” refers to the limit $c \rightarrow 0$ where c is the speed of light [Lévy-Leblond '65].
 \implies Opposite to the usual Galilean limit ($c \rightarrow \infty$).
- Carrollian limit of the Poincaré algebra:
 \implies Translations $H = \partial_t$, $P_i = \partial_i$ and rotations $J_{ij} = x_i \partial_j - x_j \partial_i$ are unchanged.
 \implies Boosts are affected: $B_i = c^2 t \partial_i - x_i \partial_t \xrightarrow{c \rightarrow 0} B_i = -x_i \partial_t$.
- Carrollian boosts shift time but do not affect space:

$$t' = t - \vec{b} \cdot \vec{x}, \quad \vec{x}' = \vec{x}$$

\implies Space becomes absolute (see diagram).

\implies Opposite to the usual Galilean limit ($c \rightarrow \infty$) where time becomes absolute:

$$t' = t, \quad \vec{x}' = \vec{x} - \vec{v}t$$

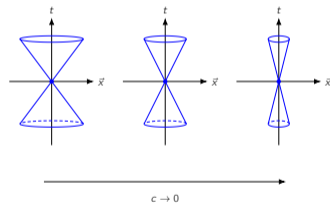
- Carrollian algebra (Inönü-Wigner contraction of Poincaré algebra when $c \rightarrow 0$):

$$[B_i, H] = 0, \quad [B_i, B_j] = 0, \quad [B_i, P_j] = \delta_{ij} H, \quad [B_k, J_{ij}] = \delta_{k[i} B_{j]}, \quad [P_k, J_{ij}] = \delta_{k[i} P_{j]}$$

- (Global) conformal Carrollian algebra (Inönü-Wigner contraction of the conformal algebra $SO(d, 2)$ when $c \rightarrow 0$):

\implies Add the dilatation: $D = (t \partial_t + x^i \partial_i)$, and the Carrollian special conformal generators: $K = x^2 \partial_u$ and

$$K_i = x^2 \partial_i - 2x_i x^j \partial_j - 2x_i t \partial_t.$$



Celestial symmetries and twistor space

- Action of $Lw_{1+\infty}$ at \mathcal{S} ? \implies Can be deduced from Carrollian OPEs.

- Define the (outgoing) soft operators [Guevara-Himwich-Pate-Strominger '21]

$$H_J^k \equiv \lim_{\Delta \rightarrow k} (\Delta - k) \Gamma(\Delta - 1) (-i)^\Delta \int_{-\infty}^{\infty} du u^{1-\Delta} \partial_u \Phi_J(u, z, \bar{z}), \quad k = 1, 0, -1, -2, \dots$$

- Action of $Lw_{1+\infty}$ on gravitons [Mason-Ruzziconi-Yellespur Srikant '23]:

$$H_2^k(z_1, \bar{z}_1) \partial_{u_2} \Phi_2(u_2, z_2, \bar{z}_2) \sim -\frac{\kappa_{2,2,-2}}{z_{12}} \sum_{m=0}^{1-k} \frac{\bar{z}_{12}^{m+1}}{m!} \frac{(-iu_2)^{1-k-m}}{(1-k-m)!} (-i\partial_{u_2})^{2-m} \partial_{\bar{z}_2}^m \Phi_2(u_2, z_2, \bar{z}_2).$$

\implies Local for $k \geq -1$ corresponding to universal soft theorems.

- The action of $Lw_{1+\infty}$ on twistor space is local and has a geometric interpretation (Poisson diffeomorphisms). [Adamo-Mason-Sharma '21] [Bu-Heuveline-Skinner '22] [Mason '22]
- Natural to relate Carrollian CFT at \mathcal{S} and twistor space:

$$\Phi_J(u, \lambda, \tilde{\lambda}) = \partial_u^{|J|} \int d^2\mu \delta(u - [\mu\tilde{\lambda}]) f(\lambda_\alpha, \mu^{\dot{\alpha}})$$

where we introduced homogeneous coordinates $(u, \lambda_\alpha, \tilde{\lambda}_{\dot{\alpha}}) = (\lambda_0 \tilde{\lambda}_{\dot{0}} u_B, \lambda_0 z_\alpha, \tilde{\lambda}_{\dot{0}} \bar{z}_{\dot{\alpha}})$.

\implies Upcoming work: what is the structure preserved by $Lw_{1+\infty}$ at \mathcal{S} ?