The Erwin Schrödinger International Institute for Mathematical Physics

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ESI NEWS

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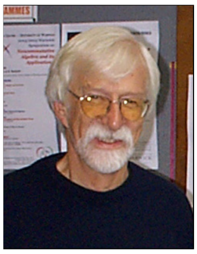
Editorial

Klaus Schmidt



This summer saw the deaths of two eminent physicists who had had close links with the ESI over many years and to whom the ESI remains grateful for their friendship over many years.

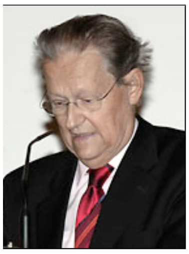
WOLFGANG KUMMER, Professor for Theoretical Physics at the Vienna University of Technology (VUT), was a member of the 'Vorstand' (Governing Board) of the Erwin Schrödinger Institute from 1993 until 2005 and was elected Honorary Member of the Institute when he resigned from the board in 2005. He was instrumental in encouraging and maintaining scientific links between the ESI and the VUT by co-organizing scientific programmes at the ESI and co-hosting ESI Senior Research Fellows on several occasions. The ESI owes him many valuable suggestions, constructive criticism and scientific stimulation.

JULIUS WESS was a key participant in the workshop *Interfaces between Mathematics and Physics* in Vienna in May 1991 which laid the foundation for the Erwin Schrödinger Institute, both scientifically and politically. He helped to impress on the Minister for Science at that time, Erhard Busek, the desirability and, indeed, necessity of creating a research institute to provide a meeting place where scientists from Eastern Europe could interact with the international scientific community at a period of great political and financial uncertainty in the post-communist world. Julius Wess helped the ESI on a second occasion, when Walter Thirring fell seriously ill in early 1992 and Julius Wess chaired a second workshop on *Interfaces between Mathematics and Physics* in March 1992 and took charge of the negotiations with the Austrian Ministry of Science which had reached a crucial point at that stage. It is fair to say that Julius Wess made a significant contribution both to the foundation and the scientific development of the ESI.

Julius Wess was elected Honorary Member of the ESI in 2005.

Wolfgang Kummer 1935 – 2007

Joachim Burgdörfer



The Austrian physics community received unexpectedly the sad news of Wolfgang Kummer's passing away on July 15, 2007 after a long and courageous battle with cancer. Just a few days earlier, Wolfgang had given in a phone conversation with the author of these lines an up-beat assessment of a new treatment he was receiving and to which he responded well. He was looking forward to a further improvement of his condition and he made already plans to return to 'his' Institute for Theoretical Physics at the Vienna University of Technology (VUT) on a more regular basis. Sudden heart failure prevented his plans from coming true.

We mourn the loss of a great scientist, of an academic teacher and researcher of highest calibre, and of a key figure that helped shaping the scientific profile and reputation of high energy physics in Austria. He was appointed full professor of Theoretical Physics in 1968 as one of the youngest full professors in Austria and served in this capacity for 36 years until reaching emeritus status in 2004. His teaching career began even earlier when he became university assistant (or assistant professor) in 1958. Over a period of almost half a century, interrupted by several research visits abroad, Wolfgang taught theoretical physics at the VUT. He received his diploma in 1958 and his Ph.D. in theoretical physics in 1960 from VUT.

His desire to specialize in the 'modern' topics of high-energy physics and quantum field theory was, in an ironic twist, partly motivated as deliberate contrast program to the dominant research activity of his doctoral supervisor and chair of the Theoretical Physics at the VUT at that time, Walter Glaser, which was firmly rooted in classical dynamics. Glaser, who died already in 1960 at the age of 54, did not live to see the recognition his joint research with his experimental collaborator and winner of the 1988 Nobel prize Ernst Ruska would eventually receive. Glaser's theoretical contributions to electron beam optics played a crucial role in the developments of the high resolution electron microscope, as Ruska noted in his Nobel lecture.

Wolfgang Kummer received strong support and tutelage from Walter Thirring, University of Vienna, who secured for him a Ford Fellowship and brought him in contact with the high-energy physics community. Kummer joined Victor Weisskopf, then director-general of CERN, as a Ford Fellow from October 1961 to March 1962. Weisskopf invited him to come back as a CERN fellow and his scientific assistant from 1963 to 1964. Returning to Vienna, Kummer completed his habilitation ('venia legendi') from VUT in 1965. He became the founding director of the Institute for High Energy Physics of the Austrian Academy of Sciences in 1966, which he led until 1971. His appointment to the second chair of Theoretical Physics at VUT took place during this period, as a result of which his center of activity gravitated towards his Alma Mater. Among the many administrative duties he took on during his tenure were the chairmanship of the physics department from 1981 to 1987 and the directorship of the Institute for Theoretical Physics from 1995 until 2003.

Over many years Kummer represented the Austrian High-Energy Community at CERN. From 1966 to 1971 he was Austrian representative at the Council of CERN. He served as its vice president from 1980 to 1983 and as its president from 1985 to 1987. On December 26, 1985 Wolfgang became victim of a terrorist attack on Vienna airport when he suffered multiple injuries from hand grenade splinters and shrapnels. Even though his injuries were life threatening and he spent eleven days in intensive care, he quickly recovered and resumed his duty as council president within weeks. Kummer also served in numerous international organizations that set science policies. Among others, he was member of the Austrian UNESCO committee for science from 1975 to 1992 and Austrian representative at the General Assembly of the International Union of pure and Applied Sciences (IUPAP) from 1996 to 2002. He also was member of the 'Vorstand' (Governing Board) of the Erwin Schrödinger International Institute for Mathematical Physics, Vienna, founded in 1993. Kummer's foremost achievement is undoubtedly the build-up of a strong theoretical high-energy physics group covering a broad range of topics in quantum field theory and (mainly 2D) quantum gravity. Kummer made fundamental contributions to quantum gauge field theory, in particular by using ghost-free non-covariant gauge fixing. Since the early 1990's he mainly worked on two-dimensional gravity and he was unceasingly productive well beyond his official retirement in 2004.

His work received many accolades and signs of recognition. Among others he was elected full member of the Austrian Academy of Sciences in 1985. Wolfgang was awarded the Schrödinger Prize in 1988, the Walter Thirring Prize 2000 (together with L. Faddeev). He received an Honorary Doctorate from the National Academy of Sciences of the Ukraine in 2005.

Kummer's contributions to the scientific community at large were by no means limited to science. He exemplified the role of an enthusiastic teacher, of an unselfish and supporting mentor of his younger colleagues, and of a colleague of impeccable integrity. He will be remembered for his unfailing dedication to the cause of science, displayed even under adverse conditions of deteriorating health.

His many friends and close colleagues are grateful for the time we were privileged to share with him. Our thoughts are with his widow, Dr. Lore Kummer who was his supportive companion for almost half a century.

Julius Wess 1934 – 2007

Walter Thirring and Bruno Zumino

Julius Wess was an imaginative, technically strong and influential theoretical physicist. He died suddenly in Hamburg on 8 August at the age of 72.

Julius, an assistant to Walter Thirring in Vienna, went on to be a professor first at Karlsruhe University and then at the University of Munich, later becoming a director of the Max Planck Institute for Physics in Munich. He was an excellent and friendly teacher and taught many students who now have positions in universities and research institutes. He was also awarded several honorary doctorates, as well as physics prizes and medals.

Julius's scientific work was influenced strongly by the recognition that the dynamics of quantum field theories is dictated largely by symmetries. His first pioneering work was on the consequences of conformal invariance for quantum fields. He then studied the representations of $SU(3)$ for the classification of hadrons. This work was done with Thirring two years before Murray Gell-Mann and Yuval Ne'eman, but not with the same representation as in the "eightfold way". Instead, it included only the A-particle in the same representation as the nucleons; at that time it seemed too daring to include the other five particles as well, as their properties seemed to be unduly different. Working with Tom Fulton, Julius went on to formulate an $SU(6)$ theory in an attempt to unify spin and isospin in agreement with special relativity.

Julius also worked in collaboration with Bruno Zumino on the mathematical structure of anomalies in non-Abelian gauge quantum field theory. This work showed that anomalies must satisfy a consistency condition and that they give rise to interaction terms (usually called Wess-Zumino terms) which have interesting topological properties. This pioneering work has had numerous ramifications for both physics and mathematics.

Julius also wrote a number of papers on supersymmetry (SUSY) and supergravity in collaboration with Zumino. This work shows that there exist 4D, local, relativistic quantum field theories that admit a symmetry between Bose and Fermi fields and that are renormalizable in the conventional sense. SUSY implies that these theories are more convergent (for instance, have no quadratic divergencies) than generic theories and require fewer renormalization constants. It is possible to formulate SUSY extensions of the Standard Model (SM) of particle physics that do not have the difficulties of the conventional SM. These extensions imply new particles and fields, which could be found at CERNs LHC, which is due to start up in 2008. Some of these theories predict particles (e.g. neutralinos) that are candidates for the dark matter of the universe.

Julius's path took him to many places around the world, and through his lovable, unassuming manner and his contagious zest for life, he rapidly made many friends. We shall all miss him greatly.

(Reprinted from CERN Courier, November 2007.)

Reminiscences of old Friendships

Email of Bruno Zumino to Walter Thirring

Dear Walter,

As you can imagine, Julius' death was a very heavy blow for me. At the ceremony where we were both awarded the Wigner prize and medal, Lochlainn O'Raifeartaigh introduced us as the "terrible twins" because of our numerous joint successful papers. Twins is not the right word, Julius was twelve years younger than me; I suggested "loving brothers" as more appropriate. Besides the Wigner prize we shared the Heineman prize and we were both awarded the Max-Planck-Medal although in different years. We also received together the Humboldt Research Award.

As you know, Julius and I met first in Vienna during the time I spent there at your invitation. Our friendship and our collaboration started there and continued in many different places, mostly in Karlsruhe, which I visited numerous times and at CERN where I was a staff member and Julius a frequent visitor.

Of my time in Vienna, I remember with great pleasure having the opportunity of meeting Erwin Schroedinger at his house. I am a great admirer of Schrödinger's work and very impressed by the breadth and the depth of his insights. His influence in physics and biology has been fantastic. He was very pleasant and gentle with me during my visit with him.

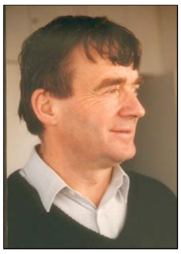
I hope that you are well and send you my very best wishes. You have always been a very good friend to me, I have not forgotten it.

Bruno Zumino

In memoriam Julius Wess

5.12.1934, Oberwölz – 8.8.2007, Hamburg

Harald Grosse



It came as a shock when we learnt that Julius Wess is not with us any longer. A great scientist, a very close friend passed away and we are left behind with so many memories of scientific discussions and co-operations and very personal relations with this extraordinary, deeply human scholar. His scientific achievements had been acknowledged by important awards: the Gottfried Wilhelm Leibniz Prize in 1986, the Max Planck medal in 1987 and the Wigner medal in 1992 (together with Bruno Zumino).

Julius had already left Vienna at the time of my own studies at the Institute of Theoretical Physics. I learnt about his work on conformal symmetry (which I was asked at my final exam with Walter Thirring) from the literature and I was deeply impressed by his work on the quark model and on Chiral Effective Lagrangians.

I personally met Julius for the first time at a conference in Frascati and at the Schladming Winter Schools in the early 1970s.

Born in the small Alpine village Oberwölz in Styria in 1934, Wess studied physics and mathematics at the University of Vienna. Here he was deeply impressed (besides his readings of Robert Musil's "Mann ohne Eigenschaften") by two of his academic teachers, the mathematician Johann Radon and the theoretical physicist Hans Thirring. In Vienna he also met his later wife Waltraud Riediger and a deep friendship developed. His thesis supervisor was Hans Thirring and Julius received his Ph.D. in theoretical physics in 1957.

After having finished his studies, a one year fellowship allowed him to go to CERN, where he elaborated on SU(3) symmetry properties of elementary particles, cooperating mainly with Markus Fierz. In 1959 he came back to Vienna as assistant Professor, where Walter Thirring had followed his father as chair of the Institute for Theoretical Physics. Here Julius met Boris Jacobsohn and Bruno Zumino. In 1960 Zumino invited him to take a position as a Research Associate at the New York University. During a subsequent half year visit at the University of Washington in Seattle a very close friendship with the physicists Grace and Lawrence Spruch started. In the years 1962 to 1966 Julius came back to Vienna and worked with Tom Fulton on the question of the unification of internal and Lorentz symmetry, which was answered negatively by the work of Sidney Coleman and Jeffrey Mandula.

In 1966 Wess became associate professor at the Courant Institute in New York. During his two years stay at the Courant Institute from 1966 to 1968 Kurt Symanzik and Wolfhard Zimmermann gave impressive lectures on quantum field theory, and his intensive collaboration with Bruno Zumino was continued. In 1968 Wess accepted an offer for a full professorship at the University of Karlsruhe, where he spent more than 20 years. In 1990 he left Karlsruhe to become director of the Max Planck Institute for Physics (Werner-Heisenberg-Institute) and professor at the Ludwig Maximilian University in Munich.

During his time in Karlsruhe he worked on various symmetry concepts in physics: together with Callan, Coleman and Zumino Effective Chiral Lagrangian were developed; together with Zumino in 1971 he discovered the anomalous Ward identities (cited 1735 times since then). Here for the first time the so called Wess-Zumino term appears which now plays an important role in model building (Wess-Zumino-Witten model) as well as in conformal field theory. This term serves up to this day as a prominent example of more complicated structures like gerbes and groupoids.

An absolute highlight was created by Julius and Bruno with their 1973 paper on *Supergauge Transformations in Four Dimensions*, which influenced particle physics considerably and has been cited (up to now) 1348 times.

Julius was always concerned with symmetries. They restrict the dynamics and yield conserved quantities. In all branches of physics they help to analyse physical systems. In 1973 bosonic and fermionic strings were formulated. Julius and Bruno deduced from these models space time supersymmetry: in nature we have particles with integer and half-integer spin, bosons and fermions; they behave differently under rotations and other transformations and obey different statistics. Electrons, protons and neutrons are fermions, their statistics implies for example the stability of matter. Fermions form the building blocks of matter, while bosons are the particles which are responsible for the forces between them.

The proposed supersymmetry allows in a fantastic manner to map from bosons to fermions, a transformation which has been achieved by using so called Grassmann variables, which square to zero. This enlargement of variables leads to an extension of space: besides the space and time coordinate a further part is introduced, which corresponds to this new variables. This simple step has enormous consequences: To each particle there corresponds an appropriate superpartner. Nature does not fulfil this rule at the energies we are fit to measure up to now.

This beautiful symmetry cures deficiencies of our quantum field theoretical models.

The masses of the superpartners are supposed to be high, and therefore they are not yet observed on earth. But they are expected to contribute to mass estimates of particles in the Universe and could explain missing energy.

It is a bitter irony that Julius no longer is able to follow the search for these particles at CERN when the LHC is switched on in 2008 and starts taking data. If Julius and Bruno were right with their vision of supersymmetry we will get a better understanding of the universe and its composition.

There are extensions of the standard model of particle physics which are based on supersymmetry, and furthermore supergravity might be a first step towards a formulation of quantized gravity.

Julius' main activity from 1973 till 1989 concerned Superphysics: keywords are supergauge transformations, superspace formulation, supergravity, supersymmetry breaking, on which he worked with young collaborators such as Richard Grimm, Martin Sohnius, Bert Ovrut, Jan Louis, Jonathan Bagger, Hermann Nicolai and others. Karlsruhe was a lively center and I had the pleasure to visit his Institute several times for a seminar.

His move to Munich in 1989 coincided with a change of subject: in 1991 the first paper with Bruno on his newly favored subject *Noncommutative Quantum Field Theory* was published. The mathematician Manin and others had elaborated already on quantum spaces, but Julius and Bruno first formulated a consistent deformation of the quantum hyperplane together with a differential calculus. After these first steps, differential calculi, deformed Lorentz group, respectively Poincaré group were on the agenda of the Munich group and studied jointly with Ogievetsky, Schmidke, Schlieker, Schirrmacher, Brano Jurco, Stefan Schraml, John Madore, Harold Steinacker, Peter Schupp, Michael Wohlgenannt, Paolo Aschieri and Bruno Zumino.

I had the pleasure to be invited many times to his institutes at LMU as well as at MPI. My own work was strongly influenced by Julius; he suggested, for example, to use the Seiberg-Witten map in order to study the question of renormalization of noncommutative gauge field models.

The Munich group in particular developed the deformed Standard model, and again it was Julius who wanted to connect these more abstract developments with physical predictions: certain decay processes are prohibited on classical space, but only occur on deformed spaces.

Julius was dealing with these extensions of the Standard model on deformed spaces, in order to improve the situation with problems of the old one, hoping that certain features of quantum gravity could already be taken into account.

The Erwin Schrödinger Institute for Mathematical Physics, created 14 years ago, is very grateful to Julius: He was a member of the committee initiating the institute, served as a Vice-president of the Board ('Vorstand') of the Institute and was elected Honorary Member of the ESI in 2002.

We were very happy, when he accepted our offer of a Senior Research Fellowship at the ESI four years ago. During his stay in Vienna he suffered a serious heart attack, from which he recovered astonishingly rapidly. When I visited him in the hospital his fist question concerned the LHC measurements. On the very next day he gave me two sheets sketching the first steps towards deformed Einstein gravity, which he then elaborated further together with Branislav Jurco, Peter Schupp and Paolo Aschieri when he returned to Munich.

In his own words: "We are in an ideal situation: from general ideas about the structure of as fundamental a concept as space-time we are led to a physical theory that can be tested experimentally."

Besides his scientific activities Julius started an East European Initiative in order to help these countries to keep up with the rapid developments. By chance, I was on a German committee which supported this initiative. A number of Workshops and Schools as well as a number of visits of physicists from former Jugoslavia were initiated by this exchange program.

After his retirement in Munich Julius moved to Hamburg. I was happy that two of his collaborators (Harold Steinacker and Michael Wohlgenannt) joined me in Vienna and became my collaborators.

We lost an eminent physicist, I lost an elder friend.

We will miss you, Julius.

Entanglement in many-body quantum physics

Frank Verstraete

One of the defining events in physics during the last decade has been the spectacular advance made in the field of strongly correlated quantum many body systems: the observation of quantum phase transitions in optical lattices and the realization that many-body entanglement can be exploited to build quantum computers are only two of the notable breakthroughs. In a remarkable turn of events, the tools developed in the context of quantum information science have been shown to shed a new light on the ones used to describe strongly correlated quantum many-body systems as studied in a wide variety of fields and has opened up many exciting interdisciplinary research avenues involving mathematical physics, condensed matter and atomic physics, and information and computational complexity theory.

The key ingredient that distinguishes the quantum from the classical world is the concept of entanglement. As a response to the Einstein-Podolsky-Rosen paper in 1935[1], Schrödinger coined the concept of entanglement[2] and recognized it as being *the defining characteristic of quantum mechanics*. In the early-days of quantum mechanics however, people were too busy with the many successful applications of quantum mechanics to really pay attention to such foundational issues. Things changed drastically in the 60's when John Bell, working as a high energy physicist in CERN, made the discovery that many-particle quantum states can in principle exhibit correlations that are stronger than correlations allowed for by local hidden variable models[3]. Although a loophole-free Bell experiment has still not been performed, Bell's work anticipated the fascinating quest to contrast the power of quantum versus classical information processing and was one of the main catalysts for the exceptional progress made in experimental quantum optics during the last decades. As a next logical step, visionary people like D. Deutsch, C. Bennett and P. Shor understood that *entanglement* can be exploited to do information tasks such as computing and cryptography much more efficiently than possible in a classical world. The current effort in the field of quantum information science is aimed at realizing these ideas. The question to contrast the power of classical to quantum information processing, and most notably to understand the power of quantum computers that explicitly make use of the possibility of quantum interference and the quantum superposition principle, led to an explosion of work on entanglement theory. One motivation is that this might lead to the discovery of new quantum algorithms by which quantum computers can solve computational problems that are believed to be intractable on classical computers; the most interesting algorithms that have as of today been proposed are Shor's algorithm for factoring large numbers (which turns out to be very relevant in the construction of one-way functions in cryptography) and algorithms for simulating the dynamics of many-body interacting quantum systems (originally proposed by Feynman).

How do you define entanglement? In the words of Schrödinger, a pure quantum state is entangled if and only if the whole is more than the sum of its parts. More specifically, the Hilbert space of a many-body system (where many is to be understood as largerthan 1) is a tensor product of the ones describing single particles (those can correspond to e.g. modes in Fock space, to localized spins, to the polorization of a photon, ...). A pure quantum state is called separable if and only if the global wavefunction is a product of such single-particle wavefunctions, and entangled otherwise; a mixed quantum state is called entangled iff it cannot be written as a convex sum of pure separable states. Note that the possibility of entanglement is nothing more than a direct consequence of the superposition principle. Note also that the notion of entanglement strongly depends on the choice of local Hilbert spaces: a Slater determinant is considered unentangled with relation to its normal mode decomposition, but can be highly entangled from the local point of view if these modes are delocalized.

A lot of work has been done to quantify entanglement. In the case of a bipartite (i.e. 2-particle) pure quantum system $|\psi_{AB}\rangle$, the natural measure of entanglement is the von-Neumann entropy of the local reduced density operator ρ_A or ρ_B[4]:

$$S(|\psi_{AB}\rangle) = -\mathrm{Tr}\rho_\mathrm{A}\log_2(\rho_\mathrm{A}) = -\mathrm{Tr}\rho_\mathrm{B}\log_2(\rho_\mathrm{B}).$$

In essence, this entanglement entropy quantifies the maximal amount of Shannon information that A can obtain by doing a measurement on his part about the measurement outcome of part B. An entropic criterion is desirable as the amount of entanglement becomes additive for independent copies, but there is also a deeper reason why this measure is used: it can be proven that any collection of states with a given mean entanglement can be interconverted into any other collection of states with the same mean entropy by only local operations and classical communication[5]. In essence, this means that systems with the same amount of entanglement are equally useful for distributed quantum information tasks such as quantum communication, and the entanglement entropy is therefore the unique measure that quantifies how useful entanglement is from the local point of view. Actually, it is possible to make a formal analogy between the theory of pure state entanglement and thermodynamics, in which local operations and classical operations that preserve the entanglement correspond to adiabatic processes in thermodynamics.

Entanglement appears everywhere in quantum mechanical systems, and there are many complementary viewpoints on it. From the point of view of quantum information theory, it is a resource that allows for revolutionary information theoretic tasks such as quantum computation and quantum cryptography (without entanglement, a quantum computer would not be more powerful than a classical one). From the point of view of quantum many-body physics, entanglement gives rise to quantum phase transitions and exotic new phases of matter exhibiting e.g. topological quantum order (i.e. a nonlocal order parameter) such as occurring in the fractional quantum Hall effect. From the point of view of the numerical simulation of strongly correlated quantum systems such as quantum spin systems and also appearing in computational quantum chemistry, entanglement is the enemy number one as it makes simulation so hard. Of course, these viewpoints are mutually compatible: the complexity of simulating entangled quantum systems is intimately connected to the power of quantum computation; the possibility of topological quantum order turns out to be strongly related to the notion of quantum error correction. It is this interplay between those complementary viewpoints that makes the study of entanglement such a rich subject.

Recently, there has been much interest in investigating the amount and type of entanglement that is naturally present in strongly correlated quantum systems. On the one hand, this was motivated by the question of whether the amount and type of entanglement needed to do quantum computation could be present in the ground-state wave functions of quantum spin systems. On the other hand, the hope is that the study of entanglement in strongly correlated quantum systems could elucidate the underlying structure of the associated wavefunctions, which on its turn might lead to new ways of simulating them.

Concerning the first question, a local 5-body quantum spin 1/2 (qubit) Hamiltonian on a square lattice was identified whose ground state is a so–called cluster state and allows for any (i.e. universal) quantum computation by doing adaptive local single-qubit measurements on it[6]. This was a surprising result as it showed that ground states of local 2-D quantum spin models contain enough entanglement for doing universal quantum computation (note that local one-qubit operations can never create entanglement). The associated Hamiltonian is unusual for a quantum Hamiltonian as it consists of a sum of local commuting terms; this means e.g. that local perturbations will never spread by virtue of Hamiltonian evolution. It turns out that these cluster states and the way to do quantum computation with them can be understood within the formalism of valence bond states or projected entangled pair states (PEPS)[7]. This class of states plays also a central role in the context of simulation of quantum spin systems, and we will later come back to them.

Concerning the question about the nature of entanglement in many-body systems, we would like to get a better understanding on the nature of the wavefunctions present in ground states of strongly correlated quantum systems. The study of correlations, both quantum and classical, is an very rich field and lies at the heart of many of the most exciting discoveries in the fields of statistical physics and quantum information theory: quantum phase transitions occur due to the appearance of long-range correlations, and the theory of entanglement is all about quantifying the amount of quantum correlations and might lead to a better understanding of the emergence of collective phenomena. The natural choice to quantify correlations in a quantum spin system endowed with a metric is to look at the connected correlation functions

$$\langle O_A O_B\rangle - \langle O_A\rangle\langle O_B\rangle$$

as a function of the distance between two regions A and B. Non-critical systems exhibit exponentially decaying correlation functions, leading to the definition of a correlations length. Despite the basic nature of this result, it has only been proven very recently[8]; the main technical ingredient for that proof was the use of the so-called Lieb-Robinson bound on the velocity by which correlations spread with respect to local Hamiltonian evolution[9]. The notion of a correlation length is very fundamental and quantifies the amount of degree of localization of the relevant degrees of freedom in the system. Intuitively, the notion of a correlation length should set the length scale at which "the whole becomes equal to the sum of its parts"; in other words, if the distance between A and B becomes much longer than the correlation length, we should have $\rho_{AB} \simeq \rho_A \otimes \rho_B$. However, just looking at 2-point correlation functions can be problematic: there exist quantum states ρ_{AB} for which all two-point correlation functions are arbitrarily small, and nevertheless they can be proven to be arbitrarily far from product states[10].

In the case of zero-temperature quantum systems, another obvious choice for quantifying the amount of correlations is to calculate the entanglement entropy of a region A of spins as a function of the size and shape of the region (see Figure 1 below). This turns out to be interesting as it will lead to useful insights into the nature of the associated wavefunctions. It has been conjectured that the entanglement entropy for noncritical systems obeys an area law, i.e. scales as the size of the boundary $|\partial A|$, indicating that the only correlations that are relevant are the ones around that boundary[11]. It has been shown that this area law is violated mildly for critical one-dimensional quantum spin and critical two-dimensional free fermionic systems, for which a multiplicative logarithmic correction has to be added[12]

$$S(\rho_A) \simeq |\partial A| \log(|\partial A|).$$

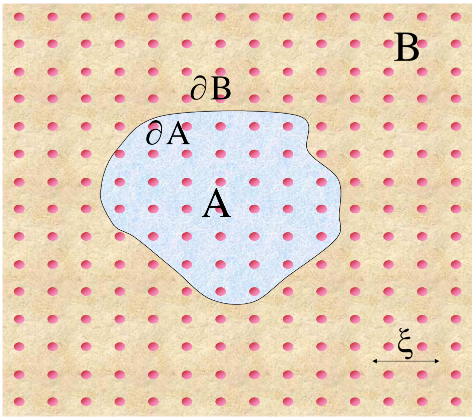


Figure 1. A quantum spin system on a lattice: the lattice is divided into regions A and B with borders ∂A and ∂B, respectively. In the case of ground states of local Hamiltonians, the entanglement entropy between the regions A and B scales like the area ∂A, as opposed to the volume $|A|$.

However, there are also critical quantum spin systems known in 2 dimensions for which a strict area law holds[13], and hence it is in general an open question of how to relate area laws to the notion of a correlation length.

The situation is much clearer in the case of finite temperature systems in thermal equilibrium. In that case, correlations can be quantified by using the concept of mutual information:

$$I(A:B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}).$$

The mutual information is zero if and only if the state is a product state $\rho_A \otimes \rho_B$ and has the same operational meaning as the entanglement entropy (actually, it is equal to twice the entanglement entropy for a pure state). Very recently, a bound on this mutual information, valid both for classical and quantum thermal states of local Hamiltonians of the form $\rho_{AB} = \exp(-\beta H)/\mathrm{Tr}\exp(-\beta H)$, has been derived[14]. The argument works equally well for spin systems, for systems with infinite dimensional local Hilbert spaces such as bosons, and for fermionic systems. The proof is short enough to reproduce here. Consider two regions A and B, and write H as a sum of 3 terms H_A, H_B, H_∂; H_A, H_B contains the terms of the Hamiltonian that acting only on A, B, and H_∂ contains the terms that represent the interactions between across the boundary. Thermal states are variationally characterized by the fact that they minimize the free energy; hence, any other state has a higher free energy, and in particular $\rho_A \otimes \rho_B$ (here, ρ_A, ρ_B are the reduced density operators of the global thermal state ρ_{AB}). We hence have:

$$\mathrm{Tr}\left(\mathrm{H}\rho_{\mathrm{AB}}\right) - \mathrm{TS}(\rho_{\mathrm{AB}}) \leq \mathrm{Tr}\left(\mathrm{H}\rho_{\mathrm{A}} \otimes \rho_{\mathrm{B}}\right) - \mathrm{TS}(\rho_{\mathrm{A}} \otimes \rho_{\mathrm{B}})$$

which is equivalent to

$$I(A:B) \leq \frac{1}{T}\mathrm{Tr}\left(\mathrm{H}_\partial \rho_{\mathrm{A}} \otimes \rho_{\mathrm{B}}\right) \leq \frac{\|\mathrm{H}_\partial\|}{\mathrm{T}}.$$

The right hand side obviously scales like the boundary of region A as opposed to its volume, and this proves that any classical and quantum thermal state exhibits an exact area law at any non-zero temperature, even in the critical case. An intriguing open problem is to connect this behaviour to the zero-temperature behaviour where logarithmic corrections arise in the case of critical quantum spin chains.

Those are laws prove that correlations are mainly concentrated around the boundary and the entanglement entropy of a block of spins scales like its boundary as opposed to its volume. That means that there is very little entanglement in ground states of local quantum Hamiltonians: all ground states live on a small manifold in Hilbert space with relatively few entanglement. This can been exploited to come up with a variational class of wavefunctions that captures this behaviour and is still easy to simulate, and this has precisely been the program that has been successfully pursued during the last years.

In the case of 1-dimensional quantum spin systems, powerful numerical renormalization group (NRG) algorithms have been devised in the 70's by Wilson[15]; those methods were later generalized to the density matrix renormalization group (DMRG) by S. White which allows to simulate ground state properties of any spin chain[16]. Both of those methods have been extremely successful and allow to calculate correlation functions of the related systems up to very large precision. Only recently, it has become clear that both of those renormalization algorithms can be rephrased as variational methods within the class of so-called matrix product states (MPS)[17]. The class of MPS is very much related to the valence-bond AKLT models put forward by Affleck, Kennedy, Lieb and Tasaki[18] and the generalizations thereof known as finitely correlated states[19]. MPS can also be generalized to so-called projected entangled pair states (PEPS) which can be defined on any lattice in any dimensions[20].

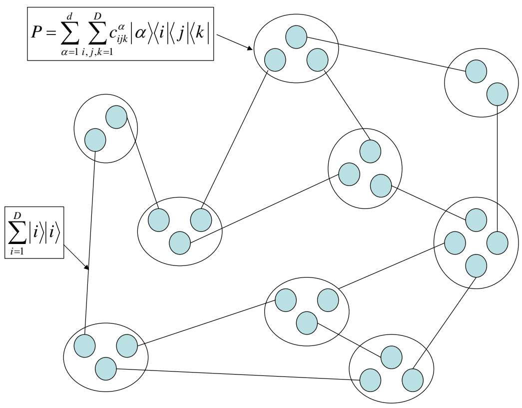


Figure 2. A PEPS can be defined on any lattice; the small circles correspond to virtual D-dimensional spins, and the map P maps them to a physical spin of dimension d represented by the bigger ellipsoids.

How can we represent those MPS and PEPS? First of all, consider a graph where the local d-dimensional spins lie on the nodes of the graph, and a collection of *virtual* bipartite entangled EPR-pairs $\sum_{i=1}^{D} |i\rangle|i\rangle$ distributed along all vertices of the graph (see figure 2). Next, we want to identify a local subspace of those *virtual* spins as the space of physical spins by applying a linear map P to them that maps c spins (c being the coordination number of the graph) to one (physical) spin of dimension d. The class of PEPS is now obtained by letting those projectors P vary over all possible $D^c \times d$ matrices. The AKLT-state is of that form, in which 2 qubits are mapped to a spin 1 state in the case of the 1-dimensional spin chain and 3 qubits to a spin $3/2$ state in the case of a hexagonal lattice. The cluster state discussed earlier is also of that form, and the quantum computation going on when doing local measurements can be understood by identifying the underlying virtual qubits as the logical qubits[21]. In the case of a 1-dimensional structure, this family of states are called Matrix Product States (MPS), and a useful aspect of them is that all correlation functions can be calculated with a computational cost that scales linearly in the number of spins and polynomially in D. This is remarkable, as the dimension of the Hilbert space scales exponentially in the number of spins, and hence the class of MPS forms a subclass for which we can calculate all properties efficiently.

By making use of entanglement theory, it has recently become clear why the numerical renormalization group methods are so successful: this is a consequence of the fact that this class of MPS is rich enough to approximate any ground state of a local gapped Hamiltonian efficiently. This implies that the manifold of ground states of all local gapped one-dimensional Hamiltonians: their ground states are well approximated by MPS, and conversely all MPS are guaranteed to be ground states of local Hamiltonians. Similar statements hold for PEPS in higher dimensions. To be more precise, let's define what we mean by good approximations. Consider a family of Hamiltonians $H_N = \sum_{i=1}^{N-1} h_{i,i+1}$ parameterized by the number of spins N and nearest neigbour interactions $h_{i,i+1}$, and associated ground states $|\psi_N\rangle$. The goal is to approximate the ground states $|\psi_N\rangle$ by a family of MPS $|\psi_N^D\rangle$ such that

$$\||\psi_N\rangle - |\psi_N^D\rangle\| \leq \epsilon$$

with ϵ independent of N. The central question is: how does D has to scale as a function of $1/\epsilon$ and N such that this relation is fulfilled? If the scaling of D is polynomial in $1/\epsilon$ and N, then it means that the ground state is represented efficiently by a MPS: indeed, the previous equation implies that the expectation value of any observable on the exact ground state is arbitrary close to the one of the MPS, that the cost of getting a better precision does only scales polynomial[22], and that all correlation functions can be calculated efficiently on $|\psi_N^D\rangle$.

The previous requirements can be met under pretty broad assumptions. First of all, it has been proven that whenever an area law is satisfied for the exact ground state, a MPS will indeed exist that approximates it well with polynomial scaling in $1/\epsilon, N$ [23]. This argument works whenever the Renyi entropy $S_\alpha(\rho) = \mathrm{Tr}(\rho^\alpha)/(1-\alpha)$ for an $\alpha < 1$ of a contiguous block of spins is bounded above by a constant times the logarithm of the size of the block. This turns out to be true for all spin chains for which this quantity has been calculated exactly, including the critical Heisenberg spin chain and the Ising Hamiltonian in a transverse magnetic field. In a related recent development, it has been proven that all gapped local Hamiltonians on a spin chain obey a strict area law in terms of the Renyi entropies[24], implying the approximability by MPS. The central technical tool used in the proof is again the Lieb-Robinson bound[25]. This provides a clear theoretical justification for the numerical renormalization group methods: they are variational methods over a class of states that is rich enough to provide an very good approximation to the exact ground states[26]. The reformulation of those methods in terms of matrix product states have opened up many new exciting possibilities and allowed for new applications such as simulating spin chains at e.g. finite temperature and out of equilibrium, to calculate gaps in quantum spin systems, and most importantly to extend the formalism of numerical renormalization group methods to higher dimensions. These methods, especially the ones in 2 dimensions, are still in development, but it has become clear that they offer crucial new insights into the structure of the wavefunctions to be found in nature. Although originally formulated on the level of quantum spin systems, it has become clear that they can also be used in the broader context of bosons, fermions and field theories. Consider e.g. fermionic lattice spin systems. This is of particular interest as the identification of the phase diagram for the 2-dimensional Hubbard model is one of the central problems in condensed matter theory. Motivated by those new developments on MPS/PEPS, it has been proven that local Hamiltonians of fermions can always be mapped to local Hamiltonians of spins[27], independent of the dimension, and therefore the whole formalism of PEPS turns out to be equally applicable to fermionic lattice systems. Those methods can equally well be used to simulate quantum lattice field theories, non-equilibrium systems, classical spin systems, and work is under way to tackle problems in quantum chemistry.

In conclusion, we have argued that entanglement theory provides fundamental new insights into the nature of strongly correlated quantum spin systems. It turns out that the amount of entanglement in ground states of quantum spin systems is surprisingly low. Under pretty general assumptions, this has allowed us to identify the manifold of wavefunctions associated to the low-energy sector of strongly correlated quantum spin systems, which on its turn can be applied to devise powerful ab initio numerical variational methods for simulating them.

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Kazhdan's Property (T)

Pierre de la Harpe

Property (T) is a rigidity property first formulated by D. Kazhdan in a three page 1967 paper whose influence has been, and still is, immense.

The first success of Kazhdan's ideas was to provide a very smart solution to an old problem (going back at least to Siegel) concerning finite generation of lattices and vanishing of Betti numbers of Riemannian symmetric spaces.

Recall that a *lattice* in a locally compact group G is a discrete subgroup Γ such that the homogeneous space G/Γ has a G–invariant probability measure. For example, the classical subject of positive definite quadratic forms on $\mathbf{R}^n$ leads to the lattice $SL_n(\mathbf{Z})$ in the simple Lie group $SL_n(\mathbf{R})$, as well as to the locally symmetric spaces $SO(n) \setminus SL(n,\mathbf{R})/\Gamma$, for appropriate finite index subgroups Γ of $SL_n(\mathbf{Z})$. Other quadratic forms and other arithmetic subjects (such as division algebras) lead to other lattices in other classical Lie groups, and more generally in reductive groups over local fields. A lattice Γ in G is *uniform* if G/Γ is compact; it is then a straightforward consequence of standard facts from general topology that Γ is finitely generated. But the finite generation of non–uniform lattices in semi-simple groups is a rather deep result (even if some particular examples such as $SL_n(\mathbf{Z})$ can be shown to be finitely generated by simple methods). It is also a crucial step for later results of the theory, such as Margulis' arithmeticity theorems.

Using methods of a completely different nature of those which were used before, Kazhdan proved in an extremely short way that any lattice in G is finitely generated when $G = G_1 \times \cdots \times G_k$ is a product of simple Lie groups G_j of real ranks at least 2. The real rank of G_j is the maximal dimension of subgroups $\mathbf{R}^\ell$ of G_j which are diagonalisable in the adjoint representation; for example the real rank of $SL_n(\mathbf{R})$ is $n-1$. Kazhdan also proved that the first Betti number of the locally symmetric space $K \setminus G/\Gamma$ is zero, where K is a maximal compact subgroup of G. (Lattices in groups of real rank one would require another discussion.) The finite generation result holds for lattices in semi–simple groups G of higher ranks defined over other local fields, for example for lattices in $SL_n(\mathbf{k}((T)))$, where $n \geq 3$ and $\mathbf{k}((T)))$ is the field of Laurent series over a finite field $\mathbf{k}$.

The main ingredient of Kazhdan's approach is the theory of *unitary representations of groups* (more precisely a small and soft part of the theory). Consider a topological group G, a Hilbert space $\mathcal{H}$, the group $\mathcal{U}(\mathcal{H})$ of its unitary operators, and a representation $\pi : G \longrightarrow \mathcal{U}(\mathcal{H})$ such that the companion mapping $G \times \mathcal{H} \longrightarrow \mathcal{H}$ is continuous. We say that π has *invariant vectors* if there exists $\xi \neq 0$ in $\mathcal{H}$ such that $\pi(g)\xi = \xi$ for all $g \in G$, and that π *almost has invariant vectors* if, for any compact subset Q of G and any $\epsilon > 0$, there exists a unit vector $\xi \in \mathcal{H}$ such that $\sup_{g\in Q} \|\pi(g)\xi - \xi\| < \epsilon$. For example, the reader can check that the representation of the additive group $\mathbf{R}$ by translations of $L^2(\mathbf{R})$ almost has invariant vectors without having invariant vectors.

A topological group G has *Kazhdan's Property (T)* if any unitary representation of G which almost has invariant vectors actually has invariant vectors. It is straightforward to check that compact groups have this property. The example above shows that $\mathbf{R}$ does not have it; similarly, locally compact groups which are abelian and non–compact (or more generally amenable and non–compact) as well as non–abelian free groups do not have Property (T). Here are remarkable results, essentially all from Kazhdan's original paper:

(i) The special linear groups $SL_n(\mathbf{K})$, $n \geq 3$, and the symplectic group $Sp_{2n}(\mathbf{K})$, $n \geq 2$ have Property (T) for any local field $\mathbf{K}$ (for example $\mathbf{K} = \mathbf{R}$). It follows that $G = G_1 \times \cdots \times G_k$ as above has Property (T).

(ii) A lattice Γ in a locally compact group G has Property (T) if and only if the group G has it.

(iii) If a locally compact group G has Property (T), then G is compactly generated and $G/\overline{[G,G]}$ is compact. In particular, a countable group Γ which has Property (T) is finitely generated and its abelianisation $\Gamma/[\Gamma,\Gamma]$ is finite.

Once again, it follows that lattices as described above are finitely generated and that locally symmetric spaces $X = \Gamma \setminus G/K$ with the appropriate condition on ranks have finitely generated fundamental group $\pi_1(X)$ and first Betti number $\dim(H_1(X,\mathbf{R})) = 0$.

Property (T) was later recognized to be equivalent to a fixed–point property (Delorme, Guichardet, Serre). More precisely, let G be a locally compact group which is σ–compact; then G has Property (T) if and only if G has the so–called *Property (FH)*, namely if and only if any continuous action of G by affine isometries of a real Hilbert space has a fixed point. On the one hand, this indicates a very strong relevance of the notion for geometry; on the other hand, the cohomological formulation of this fixed point property, namely $H^1(G,\pi) = 0$ for any orthogonal representation π of G, has proved to be useful.

The rigidity contained in Property (T) has important applications in combinatorics which go back to a short paper of Margulis (1973). Let Γ be a group generated by a finite set S; assume that Γ has Property (T) and in addition has an infinite family $(\Gamma_k)_k$ of subgroups of finite index (for example $SL_3(\mathbf{Z})$, with the kernels of the reductions $SL_3(\mathbf{Z}) \longrightarrow SL_3(\mathbf{Z}/p^k\mathbf{Z})$ modulo p^k). Let X_k denote the Cayley graph of the quotient $Q_k = \Gamma/\Gamma_k$ with respect to the image S_k of S, namely the graph with vertex set Q_k in which q and q' are connected by an edge whenever $q^{-1}q'$ is in $S_k \cup S_k^{-1}$. Then $(X_k)_k$ is a *sequence of expanders*, namely a sequence of regular finite graphs which have remarkable properties from the point of view of geometry (isoperimetric constants), spectral theory (uniform bounds on the non trivial eigenvalues of the corresponding simple random walks), and all kinds of applications in computer science (networks of computations, data organisations, computational devices, and so on). These ideas have had a very rich posterity, including quite recent work concerning graphs defined from infinite sequences of finite groups, and more generally concerning the combinatorics of finite simple groups (Alon, Bourgain, Gamburd, Kassabov, Lubotzky, Nikolov, Shalom, Wigderson, . . .).

Techniques from Kazhdan's paper, closely dependent on the theory of semisimple groups, provided only countably many examples of countable groups with Property (T). But many more examples have been later discovered, indeed uncountably many. In particular, Property (T) plays an important role in the theory of "generic groups" and "random groups" (Gromov).

In fact, Kazhdan's insight has turned out to be relevant in a large family of subjects. It has provided the solution to problems about the existence of finitely additive invariant measures going back to Lebesgue and Ruziewicz (Margulis, Rosenblatt, Sullivan, Drinfeld). In ergodic theory, ergodic actions of groups with Property (T) have been shown to be more rigid than a priori expected (Schmidt, Connes, Weiss), and the notion is presently most important for the understanding of equivalence relations (Zimmer, Furman, Hjorth). Property (T) has appropriate formulations in operator algebras (Connes, Jones, Popa, Bekka), where it has provided the key ingredient to solve several problems, one going back to Murray and von Neumann themselves: the existence of factors with fundamental groups reduced to one element (Popa). The notion has also found its way in random walks, spectral theory, the theory of algorithms, . . . , and we should probably be open for more surprising applications.

To come back to the main theme at ESI during the spring of 2007, it is striking that amenability and Property (T) are the two extreme poles of a whole range of behaviours. For example, if we restrict for simplicity the next statement to locally compact groups, a group with Property (T) is amenable if and only if it is compact; non–compact groups which have properties "in between" amenability and (T) are in some sense the most mysterious, but what we know of the two extreme types of behaviour is often a good guide for a better understanding of the general situation.

Perturbative Quantum Field Theory: Still Surprises?

Romeo Brunetti

Quantum Field Theory aims at a unifying description of nature on the basis of the principles of quantum physics and (classical) field theory. Its main success is the development of a standard model for the theory of elementary particles which describes physics between the atomic scale and the highest energies which can be reached in present experiments. It has, however, turned out to be also very important in other branches of physics, in particular for solid state physics. Its mathematical complexity is enormous and has induced many new developments in pure mathematics. In its original formulation it was plagued by divergencies whose removal by renormalization lead to fantastically precise predictions which could be verified experimentally.

A full construction of quantum field theories was possible up to now only for particular models and in too specific situations. For realistic models one still has to rely on uncontrollable approximations under which perturbation theory, which constructs the models as formal power series in the coupling constants, is the most important one.

Perturbation theory in quantum field theory has been developed as a rigorous mathematical framework in the fifties-sixties thanks to the work of Hepp, Lehmann, Symanzik, Zimmermann, Steinmann, Epstein, Glaser, Stora, Bogoliubov, Stückelberg and several others. These authors found a mathematically consistent method to construct the perturbation series of quantum field theory at all orders, thereby making mathematical sense of the recipes for renormalizations suggested before.

More recently, we experienced a renewed interest in the foundations of perturbation theory, which may come as a surprise. Two independent directions were traced. The first took place around 1996, due to Brunetti and Fredenhagen[1], and was centered around the problem of constructing quantum field theories on curved spacetimes. The other started around the end of the nineties and is due to Connes and Kreimer[2] and deals with structural insights into the combinatorics of Feynman graphs via Hopf algebras. In both cases there arise direct connections to the application of quantum field theory to physics problems. The two settings gave a lot of striking results and applications that were unforeseen before. In particular, new aspects of the renormalization group were uncovered.

In the following we summarize the highlights of the two mentioned routes:

1. Hopf algebras and renormalization

An important progress in the connection to mathematics has been obtained recently by Connes and Kreimer[3]. Their idea of using Hopf algebras in perturbation theory has led to a better mathematical understanding of the forest formula in momentum space. Kreimer's original insight originated from a study of number-theoretic properties of Feynman integrals and related the amplitudes term by term in the perturbative expansion to polylogarithms and motivic theory as well as, ultimately, to arithmetic geometry.

It turns out that Feynman graphs carry a pre-Lie algebra structure in a natural manner. Antisymmetrizing this pre-Lie algebra delivers a Lie algebra, which provides a universal enveloping algebra whose dual is a graded commutative Hopf algebra. It has a recursive coproduct which agrees with the Bogoliubov recursion in renormalization theory. While this gives a mathematical framework to perturbation theory in momentum space Feynman integrals, it also suggests to incorporate notions of perturbative quantum field theory into mathematics.

Indeed, very similar Hopf algebras have emerged in mathematics in the study of motivic theory and the polylogarithm through the works of Spencer Bloch, Pierre Deligne, Sasha Goncharov and Don Zagier. One ultimately hopes that a link can be established between number theory and quantum field theory in studying the relevant Hopf algebras and their relation in detail.

A major problem here is the understanding of the quantum equations of motion, which are governed by the closed Hochschild one-cocycles of the Hopf algebra.

This Hochschild cohomology of perturbation theory illuminates the role of locality in momentum and coordinate space approaches. At the same time, it provides a crucial input into the function theory of the polylogarithms, and certainly into a yet to be developed function theory of quantum field theory amplitudes. Extensions of these ideas to gauge theories are under active investigation, as well as the connection to motivic theory.

At the same time, Connes and Marcolli[4] are incorporating the techniques of arithmetic geometry into quantum field theories, which utilize again the underlying Hopf structure in the context of Tannakian categories, intimately connected again to the theory of the polylogarithm.

2. Epstein-Glaser perturbative approach.

Another important direction of recent research has been put forward by Brunetti and Fredenhagen[5] and refined by Hollands and Wald in a series of papers[6]. The local point of view is emphasized, via a generalization of the Epstein and Glaser approach, and allows a description of perturbation theory on any background spacetime. Basic to this approach is the connection with the field of microlocal analysis pioneered by Radzikowski. These methods allowed the cited authors to prove for the first time, that up to possible additional invariant terms of the metric, the classification of (ultraviolet) renormalization in a general spacetime follows the same rules as that on Minkowski spacetime. Actually the theory suggests further possibilities, as envisaged recently by Brunetti and Fredenhagen[7], the most important of which is a conceptually new approach to quantum gravity, at least in the perturbative sense. In this direction it is particularly important to cite that in a very recent effort, Brunetti, Dütsch and Fredenhagen, enlarged the mathematical framework a lot, by allowing also non polynomial interactions, and better clarifying the algebraic structures used in the local approach.

Other interesting directions are that taken by Dütsch and Rehren[8] for perturbation theory on AdS and connections with the quantum field theory perspectives on holography, and, more recently, Hollands has developed a new attack to the case of local and covariant pure gauge theory. The results are rather appealing and points towards a better understanding of the (redundant) mathematical structures of gauge theories.

One expects that using all these new exciting developments will be possible to address fundamental problems in cosmology, for instance, but this is still to be done.

3. Comparison and other results

At present a throughout comparison between the Hopf algebraic and the local approaches to perturbation is unfortunately lacking. The two framework use very different languages and structures and it is a challenge to see to what extent they carry the same information on the renormalization procedure. An ongoing collaboration between Bergbauer, Brunetti and Kreimer, is based on the realization that the renormalization procedure, similarly to what is done in the local approach, e.g. as extensions of n-point distributions from $M^n \setminus \cup_{k=2}^n \Delta_k$ to the full space, can also be discussed using certain tools in algebraic geometry, namely Fulton-MacPherson compactification. The hope is that this last may shed some light on the connection since the procedure of compactification embodies the Hopf algebraic combinatorics. Once established this would provide a link from the Hopf algebraic setting to the local one. Establishing a link from the local approach to the Hopf algebras may turn out to be more entertaining.

Another important point of contact is that renormalization group ideas seem to be crucial in both approaches. Other groups have pioneered different ideas, for instance by making rigorous the work of Polchinski and Wilson[9]. However, a connection between all these seemingly different perspectives is lacking and an important issue would be a comparison and attempt to find a possible unification. A first step in this direction was done by Krajewski and collaborators[10]. He showed how to use tree-like expansions and the universal Hopf algebra of rooted trees to reformulate the Wilson-Polchinski approach. In the local approach this connection has been recently discussed by Brunetti, Dütsch and Fredenhagen. There, one is able to use the Epstein-Glaser approach to discuss and compare different ideas of renormalization groups, namely, those related to Stückelberg-Petermann, to Gell-Mann and Low, and to Wilson-Polchinski, at least in the Minkowskian case. Here, one discovers that the Stückelberg-Petermann is really a group of analytic automorphisms of the local observables, that the Gell-Mann and Low approach to scaling gives only a cocycle, which is a group only in the massless case, and for the first time a direct derivation of the Flow Equation of Polchinski in the Minkowskian case. A major improvement may come from the last approach if one would be able to find a local and covariant way to impose a cut-off in the generic Lorentzian case, something that might be extremely useful in a direct attack to the perturbative Quantum Gravity.

To summarize, Perturbative Quantum Field Theory seems still extremely alive, full of new and appealing ideas that may well provide further physical insights and unforeseen connections to many partly unexplored areas of mathematics. We are confident that even more surprises are coming.

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The Interaction of Mathematics and Physics at the Turn of the twentieth Century — a Series of Lectures

Joachim Schwermer

The emergence of mathematical physics as an independent discipline at the end of the 19th century brought with it profound discussions of the foundations of both mathematics and physics as well as a fruitful cooperation between these two fields. Far-reaching concepts of modern physics and new, fundamental mathematical structures were constructed in this period. Since summer 2005 a series of lectures, entitled "History of Mathematics and Physics", at the ESI drew attention to this topic area. The talks as given in this series in the previous years have found broad interest among students, researchers and scholars and initiated a new awareness of the historical context that goes along with the sciences in question.

This is to announce two lectures in this series during the Winter Term 2007/08.

The first lecture is by **Professor Dr. Moritz Epple** (University of Frankfurt) on the topic *Beyond Metaphysics and Intuition: Felix Hausdorff's Views on Geometry*, on Wednesday, December 12, 2007, 18:00, ESI Schrödinger Lecture Hall.

Abstract: The mathematician Felix Hausdorff, who also published literary and philosophical writings under the name of Paul Mongre, was a singular figure in fin-de-sicle and early 20th century mathematical culture. His intellectual career brought together seemingly distant cultural trends such as Nietzscheanism and modernist, 'abstract' mathematics. In my talk I will try to sketch some of Hausdorff's considerations on time, space, and geometry, topics that he approached both as a philosopher and writer, and as a mathematician.

It will be seen that philosophical rather than mathematical considerations brought Felix Hausdorff to reflections on geometry and the nature of time and space in the late 1890's. Rejecting all contemporary attempts to sketch metaphysical or intuitive 'foundations' for mathematical geometry, he strongly welcomed Hilbert's axiomatic method as a tool for exploring the different possibilities to provide mathematical systems of geometrical notions which could then be compared with empirical evidence about 'space'. In the talk I will outline Hausdorff's route to the resulting 'considered empiricism' in order to compare it with certain other contemporary views on the status of geometry, such as Poincaré's and Schlick's.

The second lecture in this series will be given by **Professor Dr. Scott Walter** (Archives Henri Poincaré, University of Nancy) on *Hermann Minkowski and the Scandal of Spacetime*, on Wednesday, January 16, 2008, 17:30, ESI Schrödinger Lecture Hall.

Abstract: The ubiquity in contemporary physics of spacetime and related geometric objects belies the near-universal rejection by physicists of Hermann Minkowski's theory from its inception in November 1907 to 1911. In time, of course, spacetime came to be synonymous with Einstein's special theory of relativity, the most powerful tool for discovery in relativistic physics, and the most effective means of presenting the new dynamics. How did this change come about? Minkowski's interpretation of spacetime was initially a scandal for physicists, challenging–and eventually overturning–some of their most cherished views of the nature of physical reality. By comparing the work of Henri Poincaré, Einstein, Minkowski and others, the scandalous aspect of spacetime is brought into sharp focus, and its initial rejection more easily understood. I will argue that in this instance, formal tools played an essential role in quelling the scandal.

We hope that these two talks serve as another opportunity to bring physicists, mathematicians, and historians of science together in a single audience.

ESI News

Awards and Prizes

Hermann Kümmel Award to Frank Verstraete

Frank Verstraete has received the "Hermann Kümmel Early Achievement Award in Many-Body Physics" 2007. This prize is awarded by the International Advisory Committe of the International Conference Series on Recent Progress in Many-Body Theories. Professor Verstraete receives the award "for his pioneering work on the use of quantum information and entanglemet theory in formulating new and powerful nemerical simulation methods for use in strongly correlated systems, stochastic nonequilibrium systems and strongly coupled quantum field theories."

*Frank Verstraete will be a co-organizer of an ESI-workshop on "Tensor network methods and entanglement in quantum many-body systems", 16. bis 18. Januar 2008, and of the ESI-programme **Entanglement and correlations in many-body quantum mechanics** in August – October 2009.*

Ignaz L. Lieben Prize to Markus Aspelmeyer

The Austrian Academy of Sciences has awarded the Ignaz L. Lieben Prize 2007 to Markus Aspelmeyer "for the outstanding achievments of the young scientist in quantum optics and quantum information." This prize is the oldest prize awarded by the Academy.

Isaac Newton Medal to Anton Zeilinger

Anton Zeilinger is the first recipient of the Isaac Newton Medal awarded by Institute of Physics (IOP). Zeilinger was honoured for "his pioneering conceptual and experimental contributions to the foundations of quantum physics, which have become the cornerstone for the rapidly-evolving field of quantum information".

START Prize to Bernhard Lamel

Bernhard Lamel was awarded a START Prize by the Austrian Science Foundation (FWF) for his groundbreaking work on 'Biholomorphic Equivalence' in the theory of functions of several complex variables. The START Prize is the highest award for young scientists in Austria.

Wittgenstein Prize to Christian Krattenthaler

Christian Krattenthaler was awarded the Wittgenstein Prize by the Austrian Science Foundation (FWF) for his groundbreaking work on combinatorial problems. The Wittgenstein Prize is the most valuable and prestigious prize for scientific research in Austria.

*Christian Krattenthaler will be one of the organizers of the ESI-programme **Combinatorics and Statistical Physics** in February – June 2008.*

Current and future activities of the ESI

Thematic Programmes 2008

Combinatorics and Statistical Physics, February 1 – June 15, 2008

Organisers: M. Bousquet-Melou, M. Drmota, C. Krattenthaler, B. Nienhuis

Workshop, May 25 – June 7, 2008

Summer School, July 7 – July 18, 2008

Metastability and Rare Events in Complex Systems, February 1 – April 30, 2008

Organizers: P.G. Bolhuis, C. Dellago, E. van den Eijnden

Workshop, February 17 – February 23, 2008

Hyberbolic Dynamical Systems, May 12 – July 5, 2008

Organisers: H. Posch, D. Szasz, L.-S. Young

Workshop, June 15 – June 29, 2008

Operator Algebras and Conformal Field Theory, August 25 – December 15, 2008

Organisers: Y. Kawahigashi, R. Longo, K.-H. Rehren, J. Yngvason

Thematic Programmes 2009

Representation theory of reductive groups — local and global aspects, January 2 – February 28, 2009

Organizers: G. Henniart, G. Muic and J. Schwermer

Number theory and physics, March 1 - April 18, 2009

Organizers: A. Carey, H. Grosse, D. Kreimer, S. Paycha, S. Rosenberg and N. Yui

Selected topics in spectral theory, May 4 – July 25, 2009

Organizers: B. Helffer, T. Hoffmann-Ostenhof and A. Laptev

Large cardinals and descriptive set theory, 2 weeks in June – July 2009

Organizers: S. Friedman, M. Goldstern, R. Jensen, A. Kechris and W.H. Woodin

Entanglement and correlations in many-body quantum mechanics, August 18 – October 17, 2009

Organizers: B. Nachtergaele, F. Verstraete and R. Werner

Other Scientific Activities in 2007

ESF Workshop on Noncommutative Quantum Field Theory, November 26 – November 29, 2007

Organizer: Harald Grosse

Fourth Vienna Central European Seminar on Particle Physics and Quantum Field Theory, November 30 – December 2, 2007

Theme: Commutative and Noncommutative Quantum Field theory

Organizer: Helmuth Hüffel

EU-NCG Miniworkshop on Ergodic Theory and von Neumann Algebras, December 3 – December 14, 2007

Organizer: K. Schmidt

Spectral theory and partial differential equations, December 10 – December 21, 2007

Organizers: T. Hofmann-Ostenhof and A. Laptev

Ergodic theory — Limit theorems and Dimensions, December 17 – December 21, 2007

Organizers: F. Hofbauer and R. Zweimüller

Other Scientific Activities in 2008

Tensor network methods and entanglement in quantum many-body systems, January 16 – January 18, 2008

Organizers: F. Verstraete, G. Vidal and M. Wolf

Ab-initio density-functional studies of intermetallic compounds, January 23 – January 25, 2008

Organizer: J. Hafner

15th Anniversary of the ESI, April 14, 2008

Organizers: W.L. Reiter, K. Schmidt, J. Schwermer and J. Yngvason

Frontiers in Mathematical Biology: Mathematical population genetics, April 14 – April 18, 2008

Organizers: R. Bürger and J. Hermisson

Topics in Mathematical Physics, July 21 – July 31, 2008

Organizers: C. Hainzl, R. Seiringer and J. Yngvason

Profinite Groups, December 7 – December 20, 2008

Organizers: K. Auinger, F. Grunewald, W. Herfort and P.A. Zalesski

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