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Contents

Editorial

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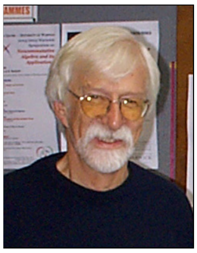

Although the symposium in commemoration of Ludwig Boltzmann's death in 1905 was only a small part of the scientific programme of the ESI in 2006, its impact extended well beyond the usual scientific clientele of the Institute.

Two public events accompanied the symposium: a *Wiener Vorlesung* for a general audience by Jürgen Renn of the MPI for History of Science in Berlin with the title *Boltzmann und das Ende des mechanischen Weltbildes*, and a Boltzmann exhibition on the premises of the ESI, organized by W. Kerber, Director of the Österreichische Zentralbibliothek für Physik. Herbert Spohn's article on *Boltzmann's Legacy* below discusses the extent to which the fundamental concepts Boltzmann formulated and struggled with are still part of current research in physics.

THE BOLTZMANN EXHIBITION AT THE ESI

Turning from the past to the present and future, but not leaving the theme of Boltzmann completely, I should mention that the Boltzmann Lecture Hall at the ESI has just been fitted with new audio-visual equipment which should bring major improvements to audibility and visibility in that room.

Another item worth mentioning is that the ESI is part of two recent successful bids for Marie Curie Research and Training Networks. Details can be found in 'ESI News' on p. 10.

Boltzmann's Legacy

Herbert Spohn

From June 7 – June 9, 2006, the ESI organized and hosted a conference under the above title to commemorate the death of Ludwig Boltzmann in Duino almost one hundred years ago on September 5, 1906. In the previous issue of ESI NEWS the reader will find a most instructive contribution by W.L. Reiter on the life and science of Boltzmann ([13]), including a substantial bibliography. To help an appreciation of the historical context, the conference was accompanied by an exhibition by the Österreichische Zentralbibliothek für Physik, organized by W. Kerber, which turned out to be highly illuminating, and which presented in particular photographs and original letters which otherwise would have remained hidden in the vaults of the library.[1]

At this conference fifteen prominent scientists covered various aspects ranging from a historical perspective to how Boltzmann's ideas are alive in present day research. Amongst them there were five Boltzmann medallists. For this the reader should know that the Commission on Statistical Physics of the IUPAP (International Union of Pure and Applied Physics) awards at each of its tri-ennial conferences two Boltzmann medals. They are regarded as the highest scientific distinction in the community of statistical physicists.

Commemorations of the kind we witnessed at this conference are common scientific practice. They provide a short moment to dispense with daily business and to reflect upon our scientific origins and at the same time envision future directions. With Boltzmann things seem to be somewhat different. Even though we are separated from him by well over a hundred years, which is a very long time span on the scale of theoretical physics, some of the problems with which Boltzmann struggled are still of interest today. This makes Boltzmann much closer to us than many of his contemporaries. (As a native speaker I may add that Boltzmann commands a very powerful language which is a mere pleasure to read. On the other hand his technical, often overlong and convoluted, papers require considerable effort.)

Many contributions by Boltzmann have become textbook material and thereby common knowledge. In contrast, in this note I plan to illustrate current research which has reasonably direct links to Boltzmann's own work. My focus will be on two key issues, one physical and one mathematical. The discussion is nontechnical and reflects, by necessity, a rather personal selection of material.

1. Emergence of macroscopic structures

1.1. *Thermal equilibrium.* It is an experimental fact that dynamical processes settle in a state of equilibrium, at least over a certain

[1] This exhibition has since been shown at the ICTP in Trieste and the Technical University of Munich. The entire exhibition can be viewed electronically at http://www.zbp.univie.ac.at/webausstellung/boltzmann/flash/boltzmann.htm.

time scale, in which matter appears to be motionless. Given the basic constituents of matter and their dynamical laws one would like to know about physical properties of such equilibrium states, such as their thermodynamic potentials, their phase diagram, perhaps also some of their fluctuation properties as expressed through correlations functions. Matter may be composed either of molecules interacting through a classical potential or of free charges interacting through the Coulomb potential. From astrophysics we know matter under more extreme conditions, as *e.g.* white dwarfs and neutron stars. High energy physics studies the quark gluon plasma as yet another manifestation of a transient equilibrium structure with its own characteristic time and length scales.

Boltzmann's contributions to the theory of equilibrium states stand out in two respects. Firstly he (and his contemporaries) had to recognize the issue and to transform it into a scientific inquiry. In demand was an atomistic model of matter. While today experimental techniques have advanced to the point where single atoms can be 'seen', in Boltzmann's time the microscopic structure of matter was a much less clear cut affair. It required a deep confidence in the mechanistic world picture to postulate the existence of atoms with their motion governed by Newton's laws. In fact, the confidence went sometimes too far. Mechanical models of the ether eventually turned out to be elusive. In any case, given that atoms move according to the laws of mechanics, how can one understand the behaviour of a large assembly of such particles? Of course, first and foremost one wants to predict the properties of thermal equilibrium states.

At this point Boltzmann argued for an answer of superb simplicity ([2]). For the purpose of computing equilibrium time averages of very particular functions on phase space (i.e. of observables with a particularly simple structure) we may pretend that the system is in a statistical state corresponding to the uniform distribution on a shell of fixed energy (and perhaps of further conserved quantities determined by the way how equilibrium is maintained) and compute the time average as ensemble average. We owe a more manageable formulation of Statistical Mechanics to J.W. Gibbs [8]. Since then it has taken generations of physicists to work out the deep implications of Boltzmann's postulate, a line of research still ongoing today. Here, somewhat generously, Quantum Statistical Mechanics can be included, since Boltzmann's postulate of the microcanonical ensemble survived the quantum revolution with rather minimal modifications.

From his ansatz Boltzmann worked out the thermal properties of ideal gases which convinced him to be on the right track. These computations sometimes generate the misconception that from the outset Boltzmann had only dilute gases in mind. His writings [2] clearly state the opposite, and Boltzmann was very well aware that the equilibrium statistical mechanics of nonideal gases would require the development of more powerful methods which in fact became available only 50 years after he had accomplished the first step.

1.2. *Dynamical evolution.* The dynamical laws governing the motion of nonviscous fluids were developed by Euler (1707 – 1783), with the dissipative corrections due to Navier (1822) and Stokes (1851). Obviously small fluid elements are assumed to satisfy Newton's second law, but a more detailed knowledge about the structure of matter is not needed. The really crucial input is the local, in space-time, validity of the laws of thermodynamics. For dilute gases, on the other hand, the mean free time is the relevant time scale, which is fine-grained enough to discern how local equilibrium is approached. The relevant space scale is set by the mean free path. A characteristic volume element then contains a huge number of atoms and some of sort of statistical analysis is asked for. Boltzmann [3] introduces the one-particle distribution function $f^{(N)}(r,v,t)$ as the central object through

$$\begin{aligned}\int_\Delta & d^3r d^3v f^{(N)}(r,v,t) \\ &= N^{-1}\cdot \big(\text{the number of particles at time } t \text{ in the volume} \\ &\qquad \Delta\subset\mathbb{R}^3\times\mathbb{R}^3,\ \text{ where the spatial scale} \\ &\qquad \text{is in units of the mean free path.}\big)\end{aligned} \tag{1}$$

Here N is the total number of particles. Note that the expression in (1) is a random variable which, however, has a variance which will tend to 0 as $N\to\infty$. Thus no averaging beyond the one expressed by the integral is needed. To write down an evolution equation for $f_t=\lim_{N\to\infty} f^{(N)}(t)$, Boltzmann assumed that in the collision of two representative molecules at location r their incoming velocities, v_1 and v_2, are statistically independent and distributed according to

$$\Big(\int_{\mathbb{R}^3} d^3v f_t(r,v)\Big)^{-2} f_t(r,v_1)f_t(r,v_2)\,. \tag{2}$$

Then

$$\frac{\partial}{\partial t}f_t(r,v) = -v\cdot\nabla_r f_t(r,v) + Q\big(f_t,f_t\big)(r,v)\,. \tag{3}$$

The first term on the right is the free streaming and the second term is the collision operator. We do not write it out explicitly and only indicate it to be quadratic in f_t, but emphasize that, in contrast to hydrodynamics, a detailed atomistic model of matter is required. In particular, the collision term contains the differential cross section as computed from the exact force law between the molecules.

The Boltzmann equation (3) generalizes with minimal changes to quantum mechanics. The free streaming results then from the semiclassical approximation to the free Schrödinger equation. In addition the differential cross section must be computed according to the laws of quantum mechanics.

From a physical point of view the reduction from the N-body dynamics to the evolution of a distribution function on the one-particle phase space is a necessity. Otherwise there would be little hope to understand in any detail the dynamical properties of dilute gases. *Closure relations*, as the one used by Boltzmann, see (2), are standard practice in many fields of physics, even through their validity is often much less transparent than in the case of dilute gases. One has to use closure relations as a working hypothesis, no luxury can be afforded. Still, from a theoretical perspective, one might wonder whether there is at least one realistic model system for which such a closure relation be can shown to be valid in a suitably specified limit, in other words up to quantifiable errors. Low density gases stand out as a paradigm where one might hope to complete such a program.

What is the problem? We consider N perfectly elastic hard balls of diameter a in a container of volume V. Their mean free path is V/Na^2, which is kept fixed while N becomes large. As for any other mechanical system we are free to impose initial conditions, say at time $t=0$. In our case they are random in such a way that (1) and (2) hold up to an error which vanishes as $N\to\infty$. This said we are no longer free to make the further assumption of independence at some later time $t>0$. Whatever the correlations may be, they are completely determined by the hard sphere dynamics. Rather, the property of independence has to be extracted from the dynamics. On the scale of the mean free time there will be order N collisions in total up to time t. So this is where the difficulty manifests itself.

O.E. Lanford [12] succeeded to control the errors in (2) by using the Bogoliubov-Born-Green-Kirkwood-Yvon hierarchy of correlation functions (he reported on his result at the conference). But the programme remains half-completed only. The uniform control of the BBGKY hierarchy could be achieved only up to 1/5-*th* of the mean free time. This in itself is a very important step because it establishes that the mathematical programme of deriving the Boltzmann equation is sound and points at the reason why (2) is valid. On the other hand, 1/5-*th* has no physical meaning. Rather it is an artefact of how the estimate on the time-dependent correlation functions is carried out. It remains as a challenge for the future to extend the result of Lanford to an arbitrary time (in units of the mean free time).

Let me add that, despite enormous efforts, for low density quantum gases (and also weakly nonlinear wave equations, which have a similar structure mathematically) an error bound in the closure relation corresponding to (2) is not available at present. There are preliminary results which make the validity of the appropriate quantum analogue for (2) very plausible (cf. [6, 1, 14]), but this constitutes only a very first step in a rigorous derivation.

2. Nonlinear evolution equations

In his famous address [9] on the occasion of the International Congress in Mathematics 1900 in Paris, D. Hilbert listed 23 problems. Problem 6 refers to what he calls the mathematical investigations of the axioms of physics. The Boltzmann equation is explicitly mentioned and Hilbert himself contributed to the subject (cf. [10]). Since then there has been a stream of attempts to elucidate the mathematical properties of the Boltzmann equation and until present days the fascination remains. (C. Villani reported on the current status during the conference.) So, how come that, even after more than 130 years since Boltzmann first wrote down his equation, the subject has not lost in momentum?

The Boltzmann equation (3) is a nonlinear evolution equation, $t \mapsto f_t$, $t \geq 0$, for the distribution function f_t, $f_t \geq 0$, which is defined on $\Lambda \times \mathbb{R}^3$ with Λ the spatial domain and $\mathbb{R}^3$ the velocity space. The equation has a peculiar dualistic form. There is the flow term which includes also the elastic collisions at the boundary of Λ. The flow term is Hamiltonian, no dissipation, and linear. Its characteristics are piecewise straight lines. On the other hand the collision operator is quadratic. It is dissipative and responsible for the approach to equilibrium. However, a collision occurs at a particular spatial point, which makes the nonlinearity rather singular. The approach to equilibrium can only be the combined effect of flow and collisions. From a physical point of view the natural space for solutions is specified by positivity, finite mass, bounded kinetic energy and bounded entropy:

$$\begin{gathered} f_t \geq 0\,, \; \int_{\Lambda\times\mathbb{R}^3} d^3r d^3v f_t(r,v) < \infty\,, \\ \int_{\Lambda\times\mathbb{R}^3} d^3r d^3v v^2 f_t(r,v) < \infty\,, \\ \int_{\Lambda\times\mathbb{R}^3} d^3r d^3v f_t(r,v) \log f_t(r,v) < \infty\,. \end{gathered} \tag{4}$$

Formally these inequalities are conserved in time. Thus the immediate mathematical problem is to establish existence and uniqueness of solutions in the space of functions defined by (4). Unfortunately such a general result is not available, except for the famous contribution of DiPerna and Lions [5] who however use a somewhat unnatural modification to the meaning of solution.

More modestly, one may want to start with spatially homogeneous solutions. Then the flow term vanishes and the collision operator is reasonably regular. Existence and uniqueness results for solutions are available. The only stationary solutions of the Boltzmann equation are the Maxwellians $M_{\rho,\beta}(v) = \rho(2\pi/\beta)^{-3/2}\, e^{-\beta v^2/2}$ (assuming that $\int d^3v v f_t(v) = 0$ for simplicity of presentation and discussion). Here $\beta = 1/k_{\mathrm{B}}T$ with T the temperature and k_{B} Boltzmann's constant. Also the mass m of a molecule is set to $m = 1$. Does f_t converge as $t \to \infty$ to $M_{\rho,\beta}$ with ρ, β fixed through the initial datum as $\int d^3v f_0(v) = \rho$, $\rho^{-1}\int d^3v v^2 f_0(v) = 3\beta^{-1}$? If so, how fast? The basic tool to elucidate the long time behaviour is the monotonicity of the Boltzmann H-function defined by

$$H(f_t) = \int d^3v f_t(v) \log f_t(v)\,. \tag{5}$$

In order to control effectively the rate of convergence one needs in addition some information on the entropy production $dH(f_t)/dt$, which then connects to the theory of logarithmic Sobolev inequalities. The convergence to equilibrium turns out to be exponentially fast, but only generically ([16]).

But even on this level basic issues remain. In quantum mechanics the Maxwell distribution is replaced by the Bose-Einstein distribution $B_{z,\beta}(v) = (z^{-1}e^{\beta v^2/2} - 1)^{-1}$, where z is the fugacity and $0 < z \leq 1$. The quantum Boltzmann equation for massive bosons has $B_{z,\beta}$ as stationary solutions. Note that their maximal density is $\rho_{\max} = \int d^3v B_{1,\beta}(v)$. But for the initial distribution function we are allowed to impose $\int d^3v f_0(v) = \rho_0 > \rho_{\max}$. So where does the excess mass go? Does the solution f_t know that the physical system wants to Bose condense, hence to form a δ-like peak at $v = 0$? If so, is there a blow up in finite time ([11])?

To return to the spatially inhomogeneous case, the folklore picture is that after a few mean free times the solution takes approximately the form of a local Maxwellian and the parameters of the Maxwellian are governed by a set of partial differential equations. This is the phenomenon which Chapman and Enskog [4] and, somewhat differently, Hilbert [10] tried to capture by their expansions. It is one of the most inspiring examples of scale separation for evolution equations, in our context the separation between the kinetic and hydrodynamic scale. Nowadays problems of such type run under the header of multiscale analysis and appear in perplexingly diverse mathematical contexts: the center manifold theory of differential equations, blow up solutions of PDEs, Whitam's equations for nonlinear hyperbolic systems, to name only a few more or less randomly.

To approximate the solution of the Boltzmann equation through the incompressible Navier-Stokes equation is under good control ([7, 15]). The compressible case is much less understood mathematically. But the hydrodynamic limit is only one facet of the long time behaviour. Does the solution of the Boltzmann equation converge to a global Maxwellian as $t \to \infty$? Through which mathematical mechanism is the local dissipation due to collisions spread throughout the system? (In brackets I recall that Hörmander's theory of hypoellipticity is designed precisely for that purpose. There one deals with a noisy dynamical system in which the noise acts only along a submanifold of the full phase space. The issue is to have conditions ensuring that nevertheless the noise induces effectively a spreading on the full phase space.) A further aspect is to impose boundary conditions at $\partial\Lambda$, the surface of Λ, in such a way that the global Maxwellian is no longer admitted as a stationary solution. Is there then another stationary solution, a nonequilibrium steady state? Do multiple steady states show up as a suitable control parameter is increased? Of course, for low-dimensional dynamical systems and for hydrodynamic systems the investigation of steady states and their associated bifurcation diagram have a long history. There is every reason to expect that the Boltzmann equation should be no exception, but the analysis becomes more difficult. Case studies remain rare.

The rich mathematical structure of the Boltzmann equation retains its fascination and has been most stimulating in the attempt to capture some of the intriguing mathematical phenomena within the realm of simplified models.

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Causes of Ecological and Genetic Diversity

Reinhard Bürger

This December (11.12.2006 – 16.12.2006), the ESI will host a workshop entitled *Causes of Ecological and Genetic Diversity* which is organized by the author and Ulf Dieckmann from IIASA, Laxenburg, Austria. In fact, this workshop is financially supported by the ESI and by the Vienna Science and Technology Fund (WWTF) via the WWTF project *Mathematics and Evolution: Mathematical and Statistical Analysis of Ecological and Genetic Diversity*. As is obvious from the title, this workshop falls into the field of mathematical biology, more precisely into (theoretical) evolutionary biology. This is not the first initiative of the ESI to support mathematical biology. From December 2002 to February 2003, the programme *Mathematical Population Genetics and Statistical Physics* was organized by Ellen and Michael Baake and by the author, with a follow-up workshop in December 2003, and in November 2004, Anton Wakolbinger, then Senior Research Fellow at the ESI, organized a workshop on *Stochastic Processes from Physics and Biology*.

To set the past and future activities into perspective, I would like to point out that mathematical biology is a vast and very heterogeneous field, both with respect to the mathematical methods as well as with respect to biological topics. Usually, the different fields are distinguished according to biological criteria. A list of the most important fields in mathematical biology includes evolutionary biology, ecology, epidemiology, game theory, demography, physiology, immunology, pattern formation, molecular biology, cell biology, biomechanics, and several fields related to medicine, e.g., cancer and tumor therapy, to mention only one. Of course, there is overlap between several of them but, by and large, they constitute quite different communities. Within most fields, a multitude of mathematical techniques and methods are employed, the most important probably being differential and difference equations (finite and infinite dimensional, ordinary and partial), stochastic processes, discrete mathematics and statistics. However, also harmonic analysis, differential geometry and other more abstract fields do have interesting and beautiful applications in mathematical biology.

Population genetics, the defining theme of the 2002/03 programme and an integral part of the upcoming workshop, is part of evolutionary biology. It is concerned with the study of the genetic composition of populations. This composition may be changed by segregation, selection, mutation, recombination, breeding structure, migration, and other genetic, ecological and evolutionary factors. Therefore, in population genetics these mechanisms and their interactions and evolutionary consequences are investigated.

Traditionally, population genetics has been applied to animal and plant breeding, to human genetics and more recently to ecology and conservation biology. It has important interfaces with molecular biology. One of the central topics is the investigation of the mechanisms that generate and maintain genetic variability in natural populations and the study of how this genetic variation, shaped by environmental influences, leads to evolutionary change, adaptation and speciation. In particular, population genetics provides the basis for understanding the evolutionary processes that have led to the diversity of life we encounter and admire.

Of paramount importance in mathematical population genetics and, more generally, in mathematical biology are models. In contrast to theoretical physics, no useful general, formal theories have been developed in the various fields of theoretical biology and, presumably, don't even exist. Surely, among others, we have Mendel's rules that describe the transmission of the hereditary material (but even here there are exceptions) and we can formalize the consequences of selection, recombination, mutation, etc., but to understand the evolution of a trait, say body size or beak length of a bird, much more is needed. A quantitative understanding of the evolution of a trait requires knowledge of the number of genes that contribute to the trait, more generally of the underlying genetic architecture (dominance, epistasis), the mutation rates of the genes, the distribution of new mutants, the genotype-phenotype map, the effects of the environment during development, the precise selective pressure exercised by the environment (or by the experimenter in case of artificial selection), the breeding structure of the population and many more. One major problem is that only a few of these factors can be incorporated into a model without it becoming mathematically intractable. A second is that the most important ingredients vary among species as well as among traits within a species. For instance, the breeding system (monogamous or polygamous with its many subforms, or random mating *vs.* nonrandom mating) is of fundamental importance and may be different in closely related species. In addition, a species may mate randomly with respect to some traits and assortatively (of varying degree) with respect to others. A third obstacle to general theories is that many key phenomena, for instance, speciation can be driven by several (very) different processes. Therefore, there are often competing models which address the same, or closely related, questions. What is considered to be important and interesting, in many cases depends on the biological background of the investigator, e.g., whether he adheres to a genetics or an ecological perspective.

As a matter of fact, mathematical models and methods have a long history in population genetics, tracing back to Gregor Mendel, who used elementary mathematics to calculate the expected frequencies of the genes in his experiments. He had a clearly defined underlying model. Francis Galton and the biometricians, notably Karl Pearson, developed new statistical methods to describe the distribution of trait values in populations and to predict their change between generations. The foundations of modern population genetics were laid by the work of Ronald A. Fisher, J.B.S. Haldane and Sewall Wright, who reconciled Mendelism with Darwinism during the second and third decades of the twentieth century. They demonstrated that the theory of evolution by natural selection, proposed by Charles Darwin (1859), can be justified on the basis of genetics as governed by Mendel's laws. The work of Fisher, Haldane and Wright was highly mathematical for the biology of that time and was properly understood by only a small number of people. Nevertheless, their influence was enormous and they set the standards for mathematical modelling and for rigour of theoretical investigations for the subsequent decades.

The major goal of the ESI programme in 2002/03 was to foster the transfer of methods from theoretical physics, stochastics and dynamical systems to problems from biological evolution, especially population genetics. Several of the highlights have been published in the ESI preprint series; they revolved around the following topics: particle systems and genetic drift, mutation-selection balance, recombination and multilocus models. The workshop in 2004 focused primarily on stochastic processes that arise, in various guises, both in physics and population genetics and on the corresponding methodology. For more details, the reader may consult the scientific reports of the ESI for 2003 and 2004 (available from ftp://ftp.esi.ac.at/pub/Reports/2003.pdf and ftp://ftp.esi.ac.at/pub/Reports/2004.pdf, respectively).

The upcoming workshop is also devoted to problems from evolutionary biology, but it goes beyond population genetics by bringing together scientists from genetics and ecology, mainly theorists but also biologists working empirically. The workshop revolves around one of the prime targets of theoretical and empirical research in evolutionary biology, namely understanding the origin and maintenance of genetic and ecological diversity. This includes the ubiquitous variation among individuals within a single population, the widespread differences of spatially distributed populations of the same species, as well as the stunning diversity among species. The mechanisms generating and maintaining this variation are not fully understood, even less so the processes that are involved in the formation of new species ([9]).

The last decade has brought about new approaches and substantial advances, both theoretical and empirical, in explaining the maintenance of genetic variation, as well as of the origin and maintenance of ecological variation (reviewed, e.g., in [1, 4, 5]). Changing ecological conditions, newly available habitats and evolution under frequency-dependent interactions can all lead to the emergence of new ecological niches. Under such conditions, subsequent evolutionary adaptations can result in intraspecific diversity, adaptive radiation and speciation. Traditionally, two complimentary routes have been pursued in theoretical studies of diversifying evolutionary processes; one originating from mathematical ecology, the other from mathematical population genetics. Each has its specific strengths and weaknesses, underscoring the seemingly insurmountable mathematical difficulties encountered in models combining both aspects. Recently, fresh approaches have been developed in both fields that now offer the possibility for a unified treatment providing new and deeper insights. One is adaptive dynamics theory, a mathematical extension of evolutionary game theory tailored to studying long-term evolutionary dynamics driven by frequency-dependent ecological interactions (e.g., [3, 4]). The other is multilocus genetics applied to ecologically relevant quantitative traits. In particular, multilocus genetic models of quantitative traits subject to various forms of balancing selection have been developed and analyzed, providing analytical conditions for the maintenance of diversity ([8, 2, 7]). Approaches of this synthetic type promise great potential: they can be generalized and extended to address problems of long-term evolution by studying the fate of new mutations (e.g., [10, 6]) and they are applicable to central evolutionary problems such as the colonization of empty niches or the maintenance of variation in heterogeneous environments.

The major scientific challenge in this field thus consists in the development and analysis of mathematical models that incorporate enough genetics and ecology to be realistic and yet remain tractable. This will be addressed in the upcoming workshop and the inspiring atmosphere at the ESI which greatly stimulates interaction among participants may well induce significant advances.

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Arithmetically Defined Kleinian Groups

Joachim Schwermer

An orientable hyperbolic 3–manifold is isometric to the quotient of hyperbolic 3–space H^3 by a discrete torsion free subgroup Γ of the group $Iso(H^3)^0$ of orientation – preserving isometries of H^3. The latter group is isomorphic to the (connected) group $PGL_2(\mathbb{C})$, the real Lie group $SL_2(\mathbb{C})$ modulo its center $\{\pm 1\}$. Generally, a discrete subgroup of $PGL_2(\mathbb{C})$ is called a Kleinian group. The group Γ is said to have *finite covolume* if H^3/Γ has finite volume, and is said to be *cocompact* if H^3/Γ is compact.

Among hyperbolic 3–manifolds, the ones originating from arithmetically defined Kleinian groups form a class of special interest. Such an arithmetically defined 3–manifold H^3/Γ is essentially determined (up to commensurability) by an algebraic number field k with exactly one complex place, an arbitrary (but possibly empty) set of real places and a quaternion algebra D over k which ramifies (at least) at all real places of k. These arithmetic Kleinian groups fall naturally into two classes. They can be distinguished by the compactness or non–compactness of the corresponding manifold H^3/Γ, since it turns out that this quotient always has finite volume.

If the arithmetic group Γ is not cocompact in $PGL_2(\mathbb{C})$, then the defining field k is an imaginary quadratic extension field $\mathbb{Q}(\sqrt{d}),\ d < 0,\ d$ a square free integer. An arithmetic group of this type is commensurable to the group $PGL_2(\mathcal{O}_d)$ where $\mathcal{O}_d$ denotes the ring of integers in k. As early as 1892 L. Bianchi studied this class of groups, today named after him.

If the arithmetic group Γ is cocompact in $PGL_2(\mathbb{C})$, then the group Γ arises from an order in a division quaternion algebra D over k which ramifies (at least) at all real places of k.

Within Thurston's geometrization program for 3–manifolds the class of hyperbolic 3–manifolds plays a fundamental role but is still not well understood. Due to the underlying connections with number theory the arithmetically defined hyperbolic 3–manifolds seem to be in many ways more tractable. There is a fruitful interaction between geometric – topological, group – theoretical and arithmetic questions, methods and results. Many of the investigations carried through in recent years are dealt with in [4] or [13], both valuable sources.

Geometric cycles versus automorphic forms

Aside from the material covered in [13] there are some general geometric or arithmetic methods developed in the realm of the theory of arithmetic groups (in particular, those emerging in the theory of automorphic forms) which might help in understanding the specific case of arithmetically defined Kleinian groups.

In particular, from the geometric point of view, there is the concept of special cycles on arithmetic locally symmetric manifolds of the form X/Γ (e. g. hyperbolic n–manifolds H^n/Γ). These special cycles arise naturally as connected components of the fixed point set of a morphism on X/Γ induced by a rational automorphism of finite order on the underlying algebraic group. In particular, the rigorous use of non–abelian Galois cohomology serves as a suitable general framework to analyze the role these special cycles play.

This approach has the following applications in our context:

— if Γ is a Bianchi group, a study of the involution on H^3/Γ induced by the non–trivial Galois automorphism of $k/\mathbb{Q}$ shows that these non–compact manifolds admit an abundance of totally geodesic hypersurfaces. They play a fundamental role in constructing non–bounding cycles, as well as in related questions in cohomology and its interpretation in terms of the automorphic spectrum (e.g. see [5], [6], [17])

— by use of the general formula for the intersection number of special cycles phrased in terms of non–abelian Galois cohomology (as proved in [18]) one obtains a slightly alternative approach to the non–vanishing result of Millson–Raghunathan [15] for the Betti numbers of certain compact arithmetically defined hyperbolic n–manifolds. These correspond (up to commensurability) to groups of units of non–degenerate quadratic forms on l^{n+1} of index $(n, 1)$, all of whose conjugates are positive definite, and where $l \neq \mathbb{Q}$ is a totally real number field.

In the case $n = 3$, this construction determines a specific class $\mathcal{M}$ of cocompact Kleinian groups under the exceptional isomorphism

$$PGL_2(\mathbb{C}) \tilde{\rightarrow} Iso(H^3)^0 \tilde{\leftarrow} SO_0(3, 1).$$

There are some constraints on the defining field and the division algebra D, respectively.

From the arithmetic point of view, the principle of Langlands functoriality in the theory of automorphic forms makes it possible to obtain specific types of cuspidal automorphic forms on GL_2/k where k is an algebraic number field subject to certain conditions. This is a consequence of the base change lift as constructed in [10], [8]. This construction allows us (cf. [9]) to exhibit non–vanishing cuspidal cohomology classes for arithmetic subgroups (defined by congruence conditions) in $PGL_2(k)$ (up to subgroups of finite index) for the field in question. This result applies, in particular, to Bianchi groups.

Via the Jacquet–Langlands correspondence [7] between cuspidal automorphic representations for GL_2 and automorphic representations of its inner forms one obtains non–vanishing cohomology classes in cases of cocompact Kleinian groups. The class of groups dealt with contains the class $\mathcal{M}$ of cocompact Kleinian groups alluded to above but is much larger. Thus, this approach makes it even possible to construct non–vanishing cohomology classes on compact arithmetically defined hyperbolic 3–manifolds which do not admit totally geodesic hypersurfaces, that is, in cases not covered by the geometric methods discussed above.

For example, this construction of non–vanishing automorphic cohomology classes can be carried out in the case of the collection $\mathcal{H}$ of cocompact arithmetic Kleinian groups which are commensurable to groups of units of skew–Hermitian forms on quaternionic vector spaces. This latter result was also obtained by Li–Millson [11] using theta series. However, the method [9] as discussed here gives a *unified* approach to the non–vanishing results in the case of the two classes $\mathcal{M}$ and $\mathcal{H}$. In both cases the arithmetically defined hyperbolic 3–manifolds H^3/Γ are determined by a quaternion division algebra D over a field k as above where k contains a subfield of index 2. This subfield has to be a totally real field. Thus, a simple base change construction permits to exhibit non–trivial automorphic classes.

An example

We illustrate other results made possible in the automorphic framework by the following specific example. Its scope seems to reach beyond geometry.

Let k be an algebraic number field of degree $n = r_1 + 2r_2$. The sign of its discriminant is determined by the number of complex places of k, i.e. sign $(d_k) = (-1)^{r_2}$. Thus, given a cubic extension $E/\mathbb{Q}$ which has exactly one complex place its discriminant d_E is negative. Such a field is necessarily non–normal over $\mathbb{Q}$. More precisely, if $E = \mathbb{Q}(x)$, its normal closure N is a quadratic extension of E. It can be described as $N = E(\sqrt{d_E})_i$ its Galois group $G(N/\mathbb{Q})$ is isomorphic to S_3, the symmetric group in three letters.

By use of Cardan's formula for the root of a cubic polynomial $X^3 + aX^2 + bX + c$ over $\mathbb{Q}$ such cubic non–normal extensions can be easily constructed. Notice that any cubic can be reduced to the form $g = X^3 + pX + q$ by a change of variable. If the discriminant $-4p^3 - 27q^2$ of g is negative than g has a unique real root. Adjoining a root of g to $\mathbb{Q}$ gives a cubic extension E of the desired type. For example, let $E = \mathbb{Q}(x)$ where x is a root of the cubic polynomial $x^3 - x - 1$. This is the unique cubic field of discriminant -23.

Let D be a division quaternion algebra over E which ramifies at least at the real place (and one finite place) of E. Then the corresponding compact hyperbolic 3–manifolds have non–vanishing first Betti number up to covering (see below). This result gives strong evidence that the virtual Haken conjecture (or, in its stronger form, known as the virtual positive Betti number conjecture) is true for arithmetically defined hyperbolic 3–manifolds.

Virtual positive Betti number conjecture

One of the most interesting conjectures in 3–manifold theory is the one by Waldhausen [20] stated in 1968. It says: Suppose M is an irreducible 3–manifold whose fundamental group is infinite. Then there exists a finite covering M' of M which is Haken, that is, it is irreducible and contains an embedded incompressible surface. An even stronger form states (under the same assumptions) that there exists a finite covering M' with non–vanishing first Betti number $b_1(M')$. This form is called the virtual positive Betti number conjecture and usually attributed to Thurston ([3], 1.2.). The significance of the former conjecture lies in the fact that it is known that 3–manifolds which are virtually Haken are geometrizable.

As the most challenging one, the case of hyperbolic 3–manifolds has gained increasing attention in recent years. Some results confirming the conjecture in specific cases were obtained in [14], [12], by geometric techniques and in [9], [2] by an automorphic approach. Further evidence is given by the experiments described in [3]. For the analogous question in the case of hyperbolic n–manifolds we refer to [14], [11].

As discussed above, the geometric approach provides examples where this conjecture is proved. As a consequence of the automorphic approach one obtains

Theorem. Let H^3/Γ *be an arithmetically defined hyperbolic 3–manifold where* Γ *is a congruence group. Suppose that the defining field k is a cubic non–normal extension of* $\mathbb{Q}$. *Then there exists a finite covering of* H^3/Γ *with non–vanishing first Betti number.*

Nonetheless, the original conjecture remains open in a number of cases. For example, let $E = \mathbb{Q}(x)$ where x is a root of the irreducible quintic polynomial $g = X^5 - 9X + 3$ over $\mathbb{Q}$ (or take $f = X^5 - 16X + 2$). The polynomial has three real roots and two conjugate complex roots. The extension $E/\mathbb{Q}$ has degree 5 and is non–normal. It is not contained in any solvable extension. Let D be a division quaternion algebra over E which ramifies at least at the three real places (and one finite place) of E. Given an arithmetic subgroup in the units of D the virtual positive Betti number conjecture is not known to be true in this case at hand. To my knowledge the methods known so far do not apply. Any progress in the base change construction as described above improves the situation.

Implicit in our discussion in this account is the hope that the ideas described here might help in gaining a better understanding of the geometry as well as the number theory of arithmetically defined hyperbolic 3–manifolds.

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The workshop 'Automorphic Forms, Geometry and Arithmetic', to be held at the ESI from February 11 - February 24, 2007, will focus on several aspects of the theory of automorphic forms with an emphasis on the relations among the Langlands functoriality principle, automorphic L-functions, Galois representations, and questions in geometry, in particular, those regarding locally symmetric spaces. The study of arithmetically defined hyperbolic manifolds provides a valuable example for the richness of this topic.

Amenability

Vadim A. Kaimanovich

The notion of *amenability* is a natural generalization of finiteness or compactness. It was introduced in 1929 by J. von Neumann (under the straightforward German name *Messbarkeit* later changed to the more appropriate *Mittelbarkeit*, cf. the French *moyennabilité*; in 1955 M. M. Dye first called it amenability). *Amenable groups* are those groups which admit an *invariant mean* (rather than an invariant probability measure, which is the case for finite or compact groups).

Actually, the history of the subject goes back to H. Lebesgue who asked in 1904 whether or not a positive, finitely (but not countably!) additive, translation-invariant locally finite measure different from the standard Lebesgue measure exists on the real line. Later, a fundamental question of F. Hausdorff led to a general study of isometry-invariant measures and the well-known Banach–Tarski–Hausdorff paradox (it states that by using the axiom of choice it is possible to take a solid ball in 3-dimensional space, cut it up into finitely many non-measurable pieces and, moving them using only rotations and translations, reassemble the pieces into two balls of the same radius as the original). J. von Neumann showed that the dichotomy in this paradox resides in the different properties of the corresponding isometry groups.

Nowadays there are numerous other (equivalent) characterizations of amenable groups. The constructive *Reiter condition* (existence of approximately invariant sequences of probability measures on the group) is often used for verifying amenability (for instance, for the group of integers such a sequence is provided by the usual Cesaro averages). On the other hand, the most important application of amenability comes from its characterization by the *fixed point property* for affine actions of amenable groups on compact spaces (once again, for the integers this amounts to the Bogolyubov–Kryloff theorem on existence of invariant measures for homeomorphisms of compact sets). Other definitions of amenability can be given in isoperimetric terms (Følner sets), in terms of the representation theory (the weak containment property), in spectral terms (Kesten's spectral gap theorem), etc., etc.

The classical notion of an amenable group has been generalized in many directions and currently plays an important (and sometimes crucial) role in many areas, such as dynamical systems, von Neumann and C^*-algebras, operator K-theory, geometric group theory, rigidity theory, random walks, etc.

For instance, R. Zimmer was the first to notice that certain actions of non-amenable groups behave as if these groups were amenable, which in the late 70's led him to the notion of amenability for group actions, equivalence relations and foliations. Simultaneously R. Bowen and A. Vershik came up with the first examples of hyperfinite orbit equivalence relations for actions of non-amenable groups. Following Zimmer's work, A. Connes, J. Feldman and B. Weiss proved the equivalence of amenability and hyperfiniteness for discrete equivalence relations. Actually, groups, group actions, equivalence relations and foliations can all be treated in a unified way by using the notion of an *amenable groupoid* introduced by J. Renault.

Amenable groupoids (in particular, those associated with boundary actions) have been at the center of recent developments in the theory of operator algebras. For example, if a locally compact group admits an amenable action on a compact space, then its reduced C^*-algebra is exact. The question of whether or not every locally compact group admits such an action was settled negatively with a counterexample by M. Gromov. A recent theorem of N. Higson and G. Kasparov for groups, and its generalization to groupoids by J. L. Tu, show that amenable groups and groupoids satisfy the Baum–Connes conjecture, which led to a proof by N. Higson of the Novikov conjecture for any locally compact group (more generally, any locally compact groupoid) that admits an amenable action on a compact space.

These developments were the subject of several monographs (let alone numerous survey articles) on various aspects of amenability: F.P. Greenleaf, *Invariant means on topological groups and their applications* (1969), J.-P. Pier, *Amenable locally compact groups* (1984), A.L.T. Paterson, *Amenability* (1988), C. Anantharaman-Delaroche and J. Renault, *Amenable groupoids* (2000), V. Runde, *Lectures on amenability* (2002).

However, certain very basic questions about amenability are still very much open, one of the well-known examples being the question about the amenability of the Thompson group, which is the group of all orientation preserving piecewise linear homeomorphisms of the interval $[0, 1]$ onto itself such that the values of the derivative are powers of 2 and the discontinuity points are dyadic rationals.

It would be impossible to cover all the subjects connected with the notion of amenability within the framework of a single programme. The 'Amenability 2007' program at ESI will be concentrated on several interconnected research areas at the crossroads of Analysis, Algebra, Geometry and Probability, including:

- amenability of self-similar groups; relation with conformal dynamics for iterated monodromy groups of rational maps; non-elementary amenable groups;
- graphed equivalence relations and amenability; cost of equivalence relations; L^2 cohomology;
- amenable groupoids; topological amenability of boundary actions; amenability at infinity; Baum–Connes and Novikov conjectures;
- amenability and rigidity; bounded cohomology;
- quasi-isometric classification of amenable groups, in particular, of nilpotent and solvable ones; geometricity of various group properties;
- Dixmier's conjecture on characterization of amenability in terms of unitarizable representations;
- generalizations of amenability: A-T-menabilty (property of Haagerup); groups without free subgroups; superamenability;
- quantitative invariants of amenable groups: growth, isoperimetry, return probability, asymptotic entropy of random walks, etc.

The first workshop of the Amenability Programme 'Amenability beyond groups' will be held at the ESI from February 26 – March 17, 2006, and will offer minicourses by **Claire Anantharaman, Gabor Elek, Masaki Izumi, Vadim Kaimanovitch and Alain Valette.**

There will be two further workshops in this programme: 'Algebraic aspects of amenability' (*June 18 - June 30, 2007*) *and 'Geometric and probabilistic aspects of amenability'* (*July 2 - July 14, 2007*). *These two workshops will be supported in part by the Marie Curie Network on Geometric, Analytic and Ergodic Aspects of Group Theory* (*cf. p.* 10).

Interaction of Mathematics and Physics at the Turn of the Twentieth Century

Joachim Schwermer

The lecture series *Interaction of Mathematics and Physics at the Turn of the Twentieth Century*, initiated in 2005 by Senior Research Fellow Della D. Fenster and Joachim Schwermer, will be continued this autumn.

On November 16, 2006, there will be two talks. The first one is given by **Catherine Goldstein** (CNRS, Paris) and discusses *Geometry and Nature according to A.N. Whitehead*. Catherine Goldstein writes: *Whitehead's contributions to physics were written in the period between his famous work on logic with Bertrand Russell, the* Principia mathematica, *and the philosophical texts to Physical Science (1922), written when Whitehead was Professor of Applied Mathematics at Imperial College in London. Whitehead developed in particular an alternative theory to that of Einstein's general relativity, a theory based on a flat space-time, but which was able to offer at the time equivalent experimental predictions; it was rediscovered and developed by Synge and his collaborators in the 1950s. Whitehead's proposal was anchored on his refusal to identify fundamental physical concepts with variables in mathematical equations and, in his own words, 'to cramp the imagination of the physicist by turning physics into geometry. In the talk, we shall discuss both the scientific and the philosophical aspects of this proposal, compare them to those of more orthodox approaches, such as Einstein's, and use the case of Whitehead to discuss afresh the classical question of the role of mathematics in physics*.

The second talk on November 16, 2006, by **Jim Ritter** (Université Paris 8), deals with *Mathematizations of the World Picture: Mathematicians in Unified field theory 1920 – 1930*. Jim Ritter describes his lecture as follows: *It is generally admitted that general relativity inaugurated a new symbiosis between mathematics, particularly geometry, and physics. But in fact the modalities of this relationship, and even the fundamental objects put into play, were far from being uniform. I will examine this question from the point of view of two groups of mathematicians — Oswald Veblen and Tracy Thomas at Princeton and Élie Cartan and Henri Eyraud in Paris — who, in the 1920s, tried in quite different ways to use geometry to participate in Einstein's attempts to develop a single unified physical theory. In doing so I will discuss how the mathematicians conceptualized their new role in physics and how their efforts were received by physicists.*

Further talks in this series will be announced on the ESI webpage.

The ESI Senior Research Fellows Programme

Over the past three years the ESI has intensified its cooperation with the local scientific community and, in particular, with the graduate programmes of the University of Vienna and the Vienna University of Technology through establishing a *Senior Research Fellows Programme* which offers — among other things — advanced lecture courses for graduate and postgraduate students.

For the year 2006/07 the following lecture courses are planned.

Autumn 2006

Ioan Badulescu (Université de Poitiers): *Representation Theory of the General Linear Group over a Division Algebra*.

Thomas Mohaupt (University of Liverpool): *Black holes, supersymmetry and strings* (this lecture course will continue in May/June 2007).

Spring 2007

Vadim Kaimanovich (International University Bremen): *Amenability and Random Walks*.

Miroslav Englis (Academy of Sciences, Prague): *Analysis on Complex Symmetric Spaces*.

The ESI Junior Research Fellows Programme

The *Junior Research Fellows Programme* is now in its third year of operation. Funded by the Austrian Ministry of Science, it provides support for PhD students and young post-docs to participate in the scientific activities of the Institute and to collaborate with its visitors and members of the local scientific community. An important aspect of this scheme is to encourage women to enter research careers in mathematics or mathematical physics, and part of the funding of the program is specifically earmarked for this purpose.

In 2006 there were 28 Junior Research Fellows visiting the ESI as part of this programme, 7 of them women. Their visits ranged from two to six months. The ESI received a large number of applications of highly qualified post-docs for funding of extended visits but unfortunately only some of them could be covered by the Junior Research Fellow Programme. In view of the close and well-established links between the ESI and many leading Eastern European academic institutions this programme was particularly beneficial to young researchers from Eastern Europe and Russia. However, this year the countries of origin ranged from Austria, Poland, Great Britain, Hungary, Romania, Italy to Sweden and Mexico. The scientific interests of the Fellows included relations between theoretical physics and algebraic geometry, quantum stochastic calculus, area preserving maps, geometric Langlands programme and modular curves, to name a few. The presence of the Junior Research Fellows contributed significantly to the positive and dynamic atmosphere at the ESI.

The next deadline for applications for ESI Junior Research Fellowships will be April 30, 2007. A call for applications will be posted on the ESI web site in March 2007.

ESI News

On May 15, 2007 there will be a celebration of **Walter Thirring**'s 80th birthday at the ESI. Walter Thirring was the founding president of the ESI and is now its honorary president. He continues to be actively involved in scientific events at the Institute. There will be lectures by Elliott Lieb (Princeton), Wolfgang Rindler (Dallas) and Julius Wess (Munich) to mark the occasion.

The ESI is part of two successful bids for **Marie Curie Research and Training Networks**.

The first network will start in January 2007 and has the title **European Training Courses in Group Theory (Geometric, Analytic and Ergodic Aspects of Group Theory)**. It is coordinated by Hamish Short (Marseille) and will include the following activities:

- 22 – 26 January 2007, *Embeddings of metric spaces into Banach spaces*, one week conference, Lausanne [EPFL],
- 5 February – 2 March 2007, *Geometric group theory*, four week training course, CIRM [CNRS],
- 25 June – 6 July 2007, *Amenability*, two week training course, Vienna [ESI],
- January 2008, *Expanders*, one week training course, Jerusalem [HUJI],
- June 2008, *Non-positive curvature and the elementary theory of free groups*, one week training course, AAV, Crete [NKUA],
- Spring 2009, *Interactions between operator algebras, groups and geometry*, one week training course, CIRM [CNRS],
- July 2009, *Boundaries*, one week training course, Graz [TUG].

The second Network has the title **Noncommutative Geometry**, is coordinated by David Evans (Cardiff) and will run for four years. The network consists of the following institutions:

- Cardiff University, United Kingdom,
- Dublin Institute for Advanced Studies, Ireland,
- Københavns Universitet, Denmark,

- University of Southern Denmark, Denmark,
- Università degli Studi di Roma 'Tor Vergata', Italy,
- Universitetet i Oslo, Norway,
- Centre National de la Recherche Scientifique, France,
- Westfälische Wilhems-Universität Münster, Germany,
- Katholieke Universiteit Leuven, Belgium,
- Erwin Schrödinger Institute for Mathematical Physics, Austria,
- Institutul de Matematica 'Simon Stoilow' al Academiei Romane, Romania.

The unifying mathematical concept behind this programme is the use of noncommutative operator algebras and noncommutative geometry to understand singular spaces or quantum spaces, replacing classical topological or measure spaces and the associated commutative algebras of continuous or measurable functions with noncommutative algebras of operators. Bringing together groups in Europe having a common goal in pursuing the deep connections between various branches of mathematics and physics we plan to address their conjectures and problems through a training network preparing young researchers equipped to work in operator algebras and noncommutative geometry.

Current and future activities of the ESI

Thematic Programmes

Global Optimization – Integrating Convexity, Optimization, Logic Programming, and Computational Algebraic Geometry, October 1 – December 23, 2006

Organizers: A. Neumaier, I. Bomze, I. Emiris, L. Wolsey

Workshop: December 4 – December 8, 2006

Amenability, February 26 – July 31, 2007.

Organizers: A. Erschler, V. Kaimanovich, K. Schmidt

Workshop on amenability beyond groups, February 26 – March 17, 2007

Workshop on algebraic aspects of amenability, June 18 – June 30, 2007

Workshop on geometric and probabilistic aspects of amenability, July 2 – July 14, 2007

Mathematical and Physical Aspects of Perturbative Aproaches to Quantum Field Theory, March 1 – April 30, 2007

Organizers: R. Brunetti, K. Fredenhagen, D. Kreimer, J. Yngvason

Poisson Sigma Models, Lie Algebroids, Deformations and Higher Analogues, August 1 – September 30, 2007

Organizers: H. Bursztyn, H. Grosse, T. Strobl

Applications of the Renormalization group, October 15 – November 25, 2007

Organizers: H. Grosse, G. Huisken, V. Mastropietro

Combinatorics and Statistical Physics, February 1 – June 15, 2008

Organisers: M. Bousquet-Melou, M. Drmota, C. Krattenthaler, B. Nienhuis

Workshop, May 25 – June 7, 2008

Summer School, July 7 – July 18, 2008

Metastability and Rare Events in Complex Systems, February 1 – April 30, 2008

Organizers: P.G. Bolhuis, C. Dellago, E. van den Eijnden

Workshop, February 17 – February 23, 2008

Hyberbolic Dynamical Systems, May 12 – July 5, 2008

Organisers: H. Posch, D. Szasz, L.-S. Young

Workshop, June 15 – June 29, 2008

Operator Algebras and Conformal Field Theory, August 25 – December 15, 2008

Organisers: Y. Kawahigashi, R. Longo, K.-H. Rehren, J. Yngvason

Other Scientific Activities

Workshop: Complex Analysis, Operator Theory and Applications to Mathematical Physics, November 6 – November 17, 2006

Organizers: F. Haslinger, E. Straube, H. Upmeier

Workshop: Modern Methods of Time-Frequency Analysis, November 20 – November 24, 2006

Organizers: H. Feichtinger, K. Gröchenig, J. Benedetto

Workshop: Quantum Statistics, November 27 – December 1, 2006

Organizers: K. Audenaert, F. Verstraete and M. Wolf

Workshop: Causes of Ecological and Genetic Diversity, December 10 – December 17, 2006

Organizers: R. Bürger and U. Dieckmann

Workshop: Langlands Duality and Physics, January 9 – January 20, 2007

Organizers: E. Frenkel, N. Hitchin, N. Nekrasov, J. Schwermer, K. Vilonen

Workshop: Automorphic Forms, Geometry and Arithmetic, February 11 – February 24, 2007

Organizers: S.S. Kudla, M. Rapoport, J. Schwermer

Workshop: Lieb-Robinson Bounds and Applications, February 20–February 24, 2007

Organizers: F. Verstraete, J. Yngvason

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