The Erwin Schrödinger International Institute for Mathematical Physics

Boltzmanngasse 9/2
A-1090 Vienna, Austria

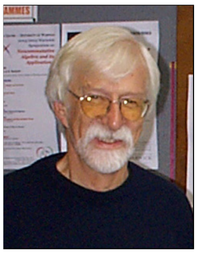
ESI NEWS

Vol. 1, Issue 1, Spring 2006

Contents

Editorial

Klaus Schmidt



The Erwin Schrödinger Institute was opened thirteen years ago in the building in the Pasteurgasse in Vienna's 9th district where Erwin Schrödinger had spent his last years. There have been quite a few changes at the ESI since that day: in 1996 the Institute moved to its present location in Boltzmanngasse 9, and during the past four years the Institute has intensified its cooperation with the University of Vienna and the Technical University of Vienna through its *Senior Research Fellows Programme*, in which the Institute offers advanced lecture courses to graduate students and postdocs at these universities. The ESI *Junior Research Fellows Programme* was initiated in 2004 and provides support for graduate students and postdocs (without restrictions on nationality) to work at the Institute for periods between 2 and 6 months. Government funding for this programme was increased last year to provide additional support for young women scientists.

The objectives of the Institute have not been affected by these changes: the promotion of research in mathematical physics and mathematics with particular emphasis on creative scientific co-operation between mathematicians and physicists.

In response to the increasingly important demand on research institutes to communicate not only with the scientific community, but also with a wider scientifically interested audience, the ESI is introducing a regular newsletter devoted to reports about its activities, new developments at the Institute and topical articles of wider interest.

Boltzmann's Legacy – The Man and his Science

Wolfgang L. Reiter

In commemoration of Ludwig Boltzmann's death in 1906 the Erwin Schrödinger Institute organizes a three-day international symposium on Boltzmann's Legacy from June 7–9, 2006, to review Boltzmann's lasting contributions to kinetic theory, thermodynamics and statistical mechanics, to present recent developments in physics and mathematics related to his work, and to investigate his influence from a historical and philosophical perspective.

The programme of the Boltzmann symposium can be found at the end of this article.

A short biography of Boltzmann

Boltzmann was born just outside the city walls of Vienna at what was then 286 Landstrasse on February 20, 1844, the very night marking the passage from Shrove Tuesday to Ash Wednesday. Boltzmann, ever the wryly ironic rationalist, commented on his birthday that he was 'born between happiness and depression'. Boltzmann was undeniably a product of some of 19th century Europe's most turbulent years: his life was marked by euphoria and depression, music and an interest in public affairs. His description of his birth date was equally fitting for the decade, something of a cultural and political watershed: in 1844 Johann Strauss the younger (1825-1899) gave his first public performance at the Dommayer Café near the Schönbrunn palace, and 1848 saw failed revolutions in Austria and the rest of Europe which ushered in a period of Habsburg absolutism after the succession of emperor Ferdinand by Franz Joseph I.

The young Ludwig was educated mainly by monks at the Akademisches Gymnasium in Linz, a provincial city in Upper Austria where his father at that time had a post as a civil servant in the state financial administration. Here he took piano lessons from Anton Bruckner (1824-1896) and developed his lifelong interest in music. The early death of his father in 1859 must have left a deep wound in the child's soul. After having passed his final school examinations, the Matura, with distinction in 1863, he enrolled at the University of Vienna to study mathematics and physics. At that time the Vienna Physics Institute was located in the same quarter where he had been born at 104 Landstrasse (now Erdbergerstrasse 15).[1] Three years later he earned his PhD which, in this era, did not require writing a doctoral thesis. At the Institute of Physics he met Josef Loschmidt (1821-1895), professor of physical chemistry and a dedicated atomist,[2] who exerted a lasting influence during his early years at the Institute. Boltzmann became his assistant in 1867. In 1868 he took the next step in his academic carrier as Privatdozent for mathematical physics, a term used for what later (around 1900) came to be called theoretical physics.

Only one year later the twenty-five year old Boltzmann was appointed professor for mathematical physics at the University of Graz, a post he held until 1873.

His first Graz period is marked by two fundamental scientific breakthroughs published in his paper *Weitere Studien über das Wärmegleichgewicht unter Gasmolekülen* (*Further studies on the thermal equilibrium of gas molecules*) of 1872, which presents his H-theorem, the very first statistical interpretation of entropy and the famous transport equation, now named after him. The H-theorem raised fierce objections, first in 1875 from his teacher and friend Loschmidt who pointed to a 'reversibility paradox', and later in 1896 by the famous mathematician Ernst Friedrich Ferdinand Zermelo (1871-1953), who pointed to the so-called 'recurrence paradox'.[3] These objections forced Boltzmann to further elaborate on his basic assumptions, finally leading him to his ground-breaking statistical interpretation of the Second Law of Thermodynamics.

In 1873 he met his future wife Henriette von Aigentler (1854-1938), the first female student at the University of Graz. (The letters of the couple during their engagement were published a few years ago by Boltzmann's grandson, Dieter Flamm.[4]) That year also brought further professional success: he accepted a post in Vienna as professor of mathematics, a major career step within the Austro-Hungarian academic hierarchy. Boltzmann stayed in Vienna only until 1876, when he went back to Graz for his second and much longer tenure as professor of physics, which lasted until 1890. In 1877 he published his probabilistic interpretation of thermodynamics, termed the 'Boltzmann Principle' by Albert Einstein (1879-1955): the entropy S of a macrostate (determined by pressure, temperature, and other variables) is related to the number W of microstates (determined by the positions and velocities of all atoms) by the famous relation $S = k \log W$ (with k the Boltzmann constant).[5] This formula is engraved on Boltzmann's tombstone at the Zentralfriedhof in Vienna.

In 1884 he proved the conjecture of his teacher Josef Stefan (1835-1893), that the total energy emitted by a black body is proportional to the fourth power of its absolute temperature.

Boltzmann's proof provided strong support for Maxwell's theory three years prior to Heinrich Hertz' experimental demonstration of the existence of electromagnetic waves. Stefan had recognized the importance of Maxwell's theory and had introduced Boltzmann, his first and most gifted student, to it. 'In the same year', to quote Carlo Cercignani, 'he also wrote a fundamental paper, generally unknown to the majority of physicists, who by reading only second-hand reports are led to the erroneous belief that Boltzmann dealt only with ideal gases; this paper clearly indicates that he considered mutually interacting molecules as well, with non-negligible potential energy, and thus [...] it is he and not Josiah Willard Gibbs (1839-1903) who should be considered as the founder of equilibrium statistical mechanics and of the method of ensembles.'[6] The year Erwin Schrödinger (1887-1961) was born Boltzmann formulated his ergodic hypothesis. By this time he was already a scientific celebrity, attracting students from abroad like the future Nobel laureates Svante August Arrhenius (1859-1927) and Walter Nernst (1864-1941), who came to Graz to study with the leading figure in thermodynamics and kinetic theory on the continent.

By 1876 Boltzmann had married Henriette, and that year he bought and refurbished an old farm house at the outskirts of Graz in Oberkroisbach, where he kept a cow to provide fresh milk for his children. Here was a man flushed with professional success, enjoying a happy family life with his wife, children and a dog. Yet clouds began to gather over this rosy Graz idyll. Several events conspired to undermine Boltzmann's position, and it was at this time that he began his long slide into depression. His beloved mother had died in 1885. In 1888 he was elected to the position of Rektor of the University of Graz and was soon confronted with a month of aggressive protests by German nationalist students. Without informing the Austrian authorities officially, Boltzmann had accepted the prestigious Berlin chair of Gustav Kirchhoff (1824-1887) and then reneged with somewhat unconvincing arguments, citing his myopia and a neglect of significant chapters of mathematical physics in Berlin. However, the most damaging episode was probably the death of his eleven year old son Ludwig in 1889 from mis-diagnosed appendicitis.

What was then termed Boltzmann's neurasthenia could no longer be ignored. Restlessness and a sort of escapism probably triggered by the feeling of an increasing isolation at Graz progressively marked his decisions. He renewed his interest in moving to Berlin, announcing his desire to leave Graz: he hoped that a change in life circumstances would calm his mental instability and insecurity. Then, after eighteen years in Graz, he accepted a chair in theoretical physics at the University of Munich in 1890. His restlessness subsided, but he was troubled by homesickness for Austria. When Stefan died in 1892 his Viennese colleagues immediately agreed to persuade Boltzmann to return to his alma mater. It took two years of hesitation, but he finally accepted the appointment in Vienna as Stefan's successor. With his worsening myopia the consideration of the generous retirement scheme offered by the University of Vienna compared to what was offered in Munich may well have been a crucial factor in his decision to return.

As it turned out, his decision to accept the chair in Vienna was a mistake. Compared with the hot bed of physical research that was Munich, his home town was distinctly provincial in this respect: he felt like a school master, training candidates in secondary school education for whom scientific work held little interest; his colleagues were less stimulating and, finally, Vienna harboured a philosophical climate strongly dominated by Ernst Mach's (1838-1916) phenomenological empiricism violently hostile to Boltzmann's atomism, the core of his life's work. When Mach moved to Vienna in 1895 to become professor of philosophy with special emphasis on the history and theory of the inductive sciences[7] both men now performed and competed for attention on the same stage as colleagues at the university and the Imperial Academy of Sciences.

After five short years in Vienna Boltzmann moved to Leipzig as Professor of Theoretical Physics, a decision obviously not taken easily because it caused a nervous breakdown, and he had to be hospitalized in a psychiatric clinic to recover. In Leipzig Boltzmann was confronted with the most energetic 'energeticist', the chemist and close adherent of Mach's philosophy, Wilhelm Ostwald (1853-1932). Leipzig was another disaster, and it was there that he made the first attempt on his life. His chair in Vienna remained vacant during his Leipzig years, probably because he already started to negotiate his return when leaving for Leipzig.

Since his unsuccessful move to Berlin, Boltzmann had become increasingly trapped in self-conflict regarding his wishes for a change of places. In 1902 he moved back to Vienna and had to promise in writing never to leave Austria again. The Emperor, not a particularly dedicated supporter of the sciences, was unamused by the peregrinations of his famous but unreliable subject. Further disruption followed: because of the urgent need for more space for the Physics Institute, the apartment that the Boltzmann family lived in at Türkenstrasse 3 was appropriated by the university.

Boltzmann's apartment was his official university residence, and to compensate him for its loss, the ministry provided him with a sum of money with which he bought a villa at Haizingergasse 26 in the suburb of Währing in the 18th district, which is still owned by his descendants. Boltzmann lectured on theoretical physics and he finished the second volume of his Vorlesungen über die Prinzipien der Mechanik. In 1904 on the occasion of his 60th birthday, he was presented with a famous Festschrift that was edited by his assistant Stefan Meyer (1872-1949) and consisted of 117 contributions from the worldwide community of physicists.[8]

'Daddy gets worse every day. He has lost his faith in our future. I had imagined a better life here in Vienna.'[9] This alarming message was written by Boltzmann's wife to their daughter Ida (1884-1910), who had remained in Leipzig to finish school. Boltzmann suffered from heavy attacks of asthma, headaches and his myopia was near to blindness. An additional heavy burden was his acceptance of the position Mach has left open after suffering a stroke in 1901, a popular lecture course on natural philosophy (*Philosophie der Natur und Methodologie der Naturwissenschaften*) which he gave in 1903 with great success in the beginning but, after a few lectures, was unable to continue.[10] This undermined his self-confidence further.

In May 1906 Boltzmann was retired from his teaching duties. Though Boltzmann's discussion of the philosophy of science and epistemology belong to his less known and little discussed legacy, he made important contributions to the field, presenting a theory of scientific change inspired by Darwin's theory of evolution. Despite his physical and mental health problems during his last years in Vienna he twice crossed the Atlantic in 1904 and 1905, having previously lectured at Clark University, Worcester, Mass., in 1899. St. Louis hosted the world fair of 1904 and Boltzmann was invited to a meeting, and in 1905 he lectured at Berkeley and Stanford.

His trip to California resulted in a most typically humorous diary *Reise eines deutschen Professors ins Eldorado*, published as part of his popular lectures in 1905 and still a delightful piece of prose.[11] But this was to be his last burst of activity, and was followed by a deep depression and hospitalization. During the winter semester of 1905/06 he gave his last course in theoretical physics. During the following term Boltzmann was unable to fulfil his teaching duties due to his physical and mental suffering. Boltzmann's mental state was officially attributed to a serious form of neurasthenia, a vaguely defined term for some sort of general weakness of the nerve system signifying a broad spectrum of conditions, ranging from anxiety to sleeplessness.[12] In the light of his symptoms it seems likely that the term 'neurasthenia' was a euphemism for a much more serious mental illness: manic depression.

Ludwig Boltzmann ended his life at one of the most scenic spots on the Adriatic coast in Duino, near Trieste, a little village overlooked by the old castle of the noble Torre e Tasso family, where Rainer Maria Rilke wrote his famous *Duino Elegies* seventeen years later. The Viennese newspaper *Die Zeit* of September 7, 1906, reported the tragic news: 'He used a short cord from the crossbar of a window casement. His daughter was the first to discover the suicide.'[13]

A day before Boltzmann's planned return to Vienna on September 6, Ludwig's wife Henriette and their youngest daughter Elsa (1891-1965) had gone for a swim after which Ludwig never rejoined them. Boltzmann had relapsed into the black state of mind that distinguished his last years. He was restless and deeply depressed despite the relaxing atmosphere of the late summer holiday by the sea side he had long promised to spend with his beloved wife. Increasingly shortsighted, anxious about his ability to perform his teaching duties as professor at the University of Vienna for the upcoming term, and beset by the manic depression which had previously led to more than one hospitalization, he ended his life at Hotel Ples, today part of the international school *Collegio del Mondo Unito*.[14] His daughter Elsa was sent back to the hotel by her mother to check up on her father and was thus the first to discover his tragic suicide, a shocking and gruesome experience she never talked about for the rest of her life.

Boltzmann passed from this world without leaving a suicide note, but the preface of his lectures on mechanics (*Vorlesungen über die Prinzipe der Mechanik*), signed Abbazia, August 3, 1897, opens with a motto carefully chosen by Boltzmann in summing up his position on science and life:

Bring' vor was wahr ist;
Schreib' so, dass es klar ist
Und verficht's, bis es mit Dir gar ist.

The funeral ceremony took place on Saturday, September 8, 1906, at the Döblinger cemetery at an unusually late hour. It was seven o'clock in the evening. The train from Trieste with Boltzmann's coffin had been delayed for hours caused by heavy weekend traffic. The last of the mourners to speak at the open grave was his assistant Stefan Meyer. He ended on behalf of Boltzmann's students with words their teacher had used: 'Reinheit und Klarheit im Kampf um die Wahrheit!'[15]

Notes

[1]Ludwig Boltzmann later wrote the following about this location of the department: 'Erdberg remained throughout my life a symbol of honest and inspired experimental work. When I succeeded in injecting a bit of life into the institute in Graz I used to call it, jokingly, 'Little Erdberg'. By this I did not mean that the available space was scarce, because it was quite ample, probably twice as much as in Stefan's institute; but I had not succeeded in equalling [sic] the spirit of Erdberg as yet. Even in Munich, when young PhDs came to tell me that they did not know what to work on, I used to think: 'How different we were in Erdberg! Today there is beautiful experimental equipment and people are looking for ideas on how to use it. We always had plenty of ideas and were only preoccupied with the lack of equipment.' Quoted in Carlo Cercignani, *Ludwig Boltzmann: The Man Who Trusted Atoms*. (Oxford, New York, Melbourne: Oxford University Press, 1998), p.6.

[2]In Loschmidt's paper of 1866, *Zur Grösse der Luftmolecüle* (*On the Size of Air Molecules*) he calculated for the first time the number of molecules per unit volume in a gas at standard temperature and pressure, which is generally known as Avogadro's number. Loschmidt's value was off by a factor of 1/30; nevertheless, it provided for the first correct estimate of the size, masses and concentrations of molecules. Ludwig Boltzmann proposed in 1899, on the occasion of the unveiling of Loschmidt's bust at the University of Vienna, that this number be called Loschmidt's number, a term that is sometimes still used in German-speaking countries.

[3]Cf. Stephen G. Brush, *Kinetic theory, Vol. 2, Irreversible processes*. (Oxford: Pergamon Press, 1966). H. Poincaré, pp. 194-202, H. Poincaré, pp. 203-207, E. Zermelo, pp. 208-217, L. Boltzmann, pp. 218-228.

[4]Dieter Flamm (Ed.), *Hochgeehrter Herr Professor! Innig geliebter Louis! Ludwig Boltzmann, Henriette von Aigentler, Briefwechsel.* (Wien, Köln, Weimar: Böhlau 1995).

[5]Ludwig Boltzmann, *Über die Beziehung zwischen dem zweiten Hauptsatze der mechanischen Wärmetheorie und der Wahrscheinlich-*

keitsrechnung, respective den Sätzen über das Wärmegleichgewicht. Sitzungsberichte der kaiserlichen Akademie der Wissenschaften, Wien, mathematisch-naturwissenschaftliche Klasse (Teil II), 76 (1877), pp. 373-435. Albert Einstein, *Über einen die Erzeugung und Verwandlung des Lichts betreffenden heuristischen Gesichtspunkt*. Annalen der Physik, 17 (1905), pp. 132-148.

[6]Carlo Cercignani, *Ludwig Boltzmann. The Man Who Trusted Atoms.* (Oxford, New York, Melbourne: Oxford University Press, 1998), p.18.

[7]The German title of Mach's lecture course was *Philosophie, insbesondere Geschichte und Theorie der induktiven Wissenschaften*.

[8]Stefan Meyer (Ed.), *Festschrift Ludwig Boltzmann gewidmet zum sechzigsten Geburtstage 20. Februar 1904* (Leipzig: J. A. Barth, 1904).

[9]Carlo Cercignani, op. cit., p.30.

[10]Ilse M. Fasol-Boltzmann (Ed.), *Ludwig Boltzmann. Principien der Naturfilosofi. Lectures on Natural Philosophy 1903-1906.* (Berlin/Heidelberg: Springer-Verlag 1990)

[11]*Ludwig Boltzmann, Populäre Schriften. Eingeleitet und ausgewählt von Engelbert Broda.* (Braunschweig/Wiesbaden: Friedr. Vieweg & Sohn 1979), pp. 258-290. Dieter Flamm (Ed.), op. cit., pp. 235-256 (translation into English in Carlo Cercignani, op. cit., pp. 231-250).

[12]Eugen Bleuler, *Lehrbuch der Psychiatrie, 14. Auflage, neubearbeitet von Manfred Bleuler*. (Berlin/Heidelberg/New York: Springer-Verlag 1979), pp. 519-522.

[13]'Aus Triest kommt uns die Meldung: Hofrat Prof. Dr. Ludwig Boltzmann, der zum Sommeraufenthalt mit seiner Tochter in Duino weilte, wurde gestern als Leiche in seinem Zimmer aufgefunden. Er hatte sich mit einem kurzen Strick am Fensterkreuz erhängt. Seine Tochter war die erste, die den Selbstmord entdeckte.' *Wiener Neuigkeiten. Selbstmord des Prof. Boltzmann.* In: Die Zeit (Vienna), Nr. 1420, 7. September 1906, p.1.

[14]Several versions have been presented for the location where Boltzmann committed suicide. The version of the mathematician and colleague of Boltzmann, Franz Mertens (1840-1927), that Boltzmann took his life in the church of Duino, can be excluded, because no re-consecration of the church is documented. Support for the version that the location has to be identified with his hotel room is given in a short notice *Die Ausführung des Selbstmords* in the Neue Freie Presse (Vienna), Nr. 15102, 7. September 1906, p.3, but without naming the hotel. The name of the hotel presented here is a private communication by Franz Gammer, Vienna.

[15]*Hofrat Prof. Boltzmann. Das Leichenbegängnis in Wien.* In: Die Zeit (Vienna), Nr. 1422, 9. September 1906, p.4. *Hofrat Prof. Boltzmann. Das Leichenbegängnis.* Neues Wiener Tagblatt (Vienna), Nr. 49, 9. September 1906, pp. 6-7.

The Scientific Programme of the Boltzmann Symposium

Wednesday, June 7, 2006

11:00 Jakob Yngvason, Universität Wien: *Boltzmann's legacy from the point of view of a mathematical physicist*

11:30 Anton Zeilinger, Universität Wien: *Boltzmann's legacy from the point of view of an experimentalist*

14:00 Joel L. Lebowitz, Rutgers University: *On the role of Boltzmann's entropy in the time evolution of macroscopic systems*

15:00 Elliott H. Lieb, Princeton University: *What if Boltzmann had known about quantum mechanics – and other questions*

16:30 David Ruelle, IHES, Bures-sur-Yvette: *Is entropy production local in infinite classical systems?*

19:30 Jürgen Renn, MPI für Wissenschaftsgeschichte, Berlin: *Wiener Vorlesung: Boltzmann und das Ende des mechanischen Weltbildes*

Thursday, June 8, 2006

09:00 Michael E. Fisher, University of Maryland: *Atoms and ions; Universality, singularity and particularity: lessons from a half-century of statistical physics*

10:00 Oscar E. Lanford III, ETH Zürich: *Deriving the Boltzmann equation from microscopic hard-sphere dynamics*

11:30 Herbert Spohn, TU München: *The Boltzmann transport-equation for weakly nonlinear lattice dynamics*

15:00 Christoph Dellago, Universität Wien: *From hard spheres to soft matter: Boltzmann's legacy for the computational physicist*

16:00 Giovanni Gallavotti, Università di Roma 'La Sapienza': *Chaoticity as a unifying principle of equilibrium and nonequilibrium statistical mechanics*

17:30 Session organized by Joel L. Lebowitz, Rutgers University: *Human rights and social responsibilities of scientists*

Friday, June 9, 2006

09:00 Peter Schuster, Universität Wien: *Boltzmann and evolution — basic questions of biology seen with atomistic glasses*

10:00 Nadine de Courtenay, CNRS, Paris: *Boltzmann's philosophy of science: a technical adventure*

11:30 Cédric Villani, ENS, Lyon: *Beyond the H-Theorem: Boltzmann's entropy in today's mathematics, from partial differential equations to geometry*

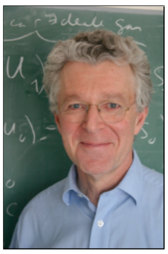
15:00 Donald S. Ornstein, Stanford University: *Random or determined?*

16:00 E.G.D. Cohen, Rockefeller University, New York: *Entropy, probability and dynamics*

Bose-Einstein Condensation

A Challenge for Mathematical Physics

Jakob Yngvason



Bose-Einstein condensation (BEC) is a macroscopic quantum phenomenon predicted theoretically by A. Einstein in 1924, when he applied to microscopic material particles some ideas about statistics of light quanta (photons) put forward by S. N. Bose shortly before. There is a certain analogy with the condensation of vapour in a vessel to a liquid when the vessel is cooled or its volume is reduced, but while that phenomenon can, in principles at least, be understood within classical statistical mechanics, Bose-Einstein condensation is a manifestation of the quantum laws of nature. Its hallmark is a complete loss of individuality of the microscopic particles when a *macroscopic* number of them settles down in a *single* quantum state. In order to appreciate this statement one must realize that there is an enormous number of different quantum states available to a microscopic particle like an atom in a vessel of macroscopic dimensions. If classical (Boltzmann) statistical physics applied the probability of a macroscopic occupation of any single one of these states would for all practical purposes be equal to zero. In quantum physics microscopic particles are of two kinds, 'bosons' or 'fermions', depending on whether their internal angular momentum ('spin') is an integer or a half integer multiple of Planck's constant. While at most one fermion can occupy a given quantum state, bosons have an opposite tendency and at sufficiently low temperatures the peculiar type of condensation mentioned with a large number of particles in the same quantum state is possible. Einstein's prediction was that BEC should take place if the temperature T and the particle density ρ satisfy the condition

$$T \leq \frac{2\pi\hbar^2}{mk}\left(\frac{\rho}{2.612}\right)^{2/3} \qquad (1)$$

where m is the mass of the particles, $\hbar$ Planck's constant and k Boltzmann's constant.

At the time Einstein made his prediction quantum mechanics was not yet fully developed and the distinction between bosons and fermions was not known. In fact, Einstein thought that his conclusions would apply to all microscopic particles, in particular electrons. This is not the case because electrons have half-integer spin and are therefore fermions. Einstein's ideas were first applied to a real physical system by F. London who suggested in 1935 that the phenomenon of superfluidity in liquid helium is related to BEC. There is, however, a very important difference between helium and the systems considered by Einstein: in liquid helium the average distance between atoms is comparable to their size and interactions can not be ignored. Einstein's considerations, on the other hand, were restricted to *ideal* gases without any interactions at all. The connection between BEC and superfluidity is, in fact, far from being simple and is still not completely understood.

The theory of interacting Bose gases and superfluidity was pioneered by N.N. Bogoliubov in seminal papers in the late 1940's. Following this work there was much theoretical activity in this field in the 50's and 60's. One of the outcomes of this research was a precise definition of the concept of BEC in the presence of interactions, formulated by O. Penrose and L. Onsager in 1956. We recall first that in quantum mechanics the state of a system of N (spinless) particles is described by complex valued function Ψ ("wave function") that is a function of the N position variables of the particles, $\vec{x}_1, \ldots, \vec{x}_N$. The function Ψ is assumed to be square integrable and normalized. For bosons it is also invariant under arbitrary permutations of the variables. The presence or absence of BEC is determined by the 'one-particle reduced density matrix', $\gamma(\vec{x}, \vec{x}')$, which is defined as

$$\gamma(\vec{x}, \vec{x}') = N \int \Psi(\vec{x}, \vec{x}_2, \ldots, \vec{x}_N) \cdot \Psi(\vec{x}', \vec{x}_2, \ldots, \vec{x}_N)^* d\vec{x}_2 \cdots \vec{x}_N.$$

It has the property that for any 1-particle state φ,

$$\bar{N}_\varphi = \int \varphi(\vec{x})^* \gamma(\vec{x}, \vec{x}') \varphi(\vec{x}') d\vec{x} d\vec{x}'$$

is the average number of particles in the 1-particle state φ if the system as a whole is in the N-particle state Ψ. The word 'average' is important here because it is in general not possible to characterize the many body state Ψ by specifying a sequence of 1-particle states, say $\varphi_1, \varphi_2, \ldots$, and assigning definite occupation numbers, $N_1, N_2, \ldots$ to each of them. This is only possible if Ψ has the special form of a symmetrized tensor product of the N_i fold tensorial powers of the φ_i. A general state Ψ, however, is not of this form because of the quantum mechanical superposition principle. Such superpositions are essential when considering interacting systems where the relevant many-body states are typically strongly entangled.

For an N-particle state Ψ with N 'large' (e.g., $N > 10^6$) we now say that Ψ exhibits BEC if for some one-particle state φ, $\bar{N}_\varphi$ is a 'sizeable fraction' (e.g., $> 1\%$) of N. The one-particle state φ_0 that maximizes $\bar{N}_\varphi$ is called the *wave function of the condensate* and the ratio $\bar{N}_{\varphi_0}/N$ is called the *condensate fraction*. The condensate fraction is in fact the ratio between the largest eigenvalue of the integral kernel $\gamma(\vec{x}, \vec{x}')$ and N so the definition can also be formulated thus: *BEC means that the one-particle density matrix has an eigenvalue of order N.*

For a mathematical treatment of BEC this 'physical' definition must be recast in the form of a statement about the asymptotic behaviour of the largest eigenvalue of γ as $N \to \infty$ while some quantity, e.g., the particle density, is kept fixed. It thus applies really to a sequence of states Ψ_N with an increasing number of variables N. The most important example is the case when Ψ_N is the ground state (the eigenstate with lowest eigenvalue) of the N-particle Hamiltonian

$$H_N = \sum_{i=1}^{N} \{-\Delta_i + V(\vec{x}_i)\} + \sum_{1 \le i < j \le N} v(\vec{x}_i - \vec{x}_j). \tag{2}$$

Here Δ_i is the Laplacian w.r.t. the variable $\vec{x}_i$, V is an external potential that confines the particles to a bounded region and v is the potential describing the mutual interactions of the particles. Natural units have been chosen so that Planck's constant and the particle mass do not appear in H_N. If the density is kept fixed as $N \to \infty$ one speaks of a *thermodynamic limit*.

Consider now a three dimensional box of side length L and let V be the potential that is zero inside the box and infinite outside. Let $\gamma_{N,L}$ be the one-particle density matrix of the ground state of N particles in the box and let $\lambda_{N,L}$ denote its largest eigenvalue. The precise *definition of BEC in the thermodynamic limit* is now as follows: There is a $c > 0$ such that

$$\lambda_{N,L} \ge cN \tag{3}$$

for all (sufficiently large) N and $L \sim N^{1/3}$ so that the density $\rho = N/L^3$ is fixed.

For an *ideal* Bose gas, i.e., if the interaction potential v vanishes, BEC in the ground state (with a condensate fraction of 100%) trivially holds. In fact, the ground state Ψ_N is in this case a tensor power $\varphi^{\otimes N}$, where φ is the ground state of the three-dimensional Laplacian in the box with Dirichlet boundary condition, and the one particle density matrix is N times the projection onto φ. In other words: All the particles are in the same one-particle state φ. Einstein's remarkable discovery was, of course, not this trivial fact. His considerations applied to *thermal equilibrium states* at *nonzero* temperatures and by his equation (1) BEC takes place in an ideal gas in the thermodynamic limit if the temperature is low enough. It is worth remarking that this result depends on the dimension of space: it is wrong in one and two dimensions.

Within the formalism of quantum statistical mechanics the definition of BEC for thermal equilibrium states is completely analogous to the one already given for ground states, and leads, for ideal gases, to Einstein's condition (1). This extension of the definition will not be elaborated further here because for interacting systems BEC is a highly nontrivial problem already in the ground state. In fact, more than 80 years after Einstein's discovery *there still exists no rigorous derivation of BEC in the thermodynamic limit from the Hamiltonian* (2) *with a realistic interaction potential v!* From a mathematical point of view BEC is a statement about the large N behaviour of a delicate spectral property of the partial differential operator H_N, namely the largest eigenvalue of the one-particle density matrix derived from its ground state or a thermal equilibrium state. The challenge for mathematical physics is to uncover this property by rigorous mathematical analysis.

On the experimental side a new era began in 1995 when the groups of W. Ketterle at MIT and of E. Cornell and C. Wieman in Boulder, Colorado, succeeded in realizing Einstein's condition (1) in a *dilute* gas of alkali atoms. 'Dilute' means here that the average particle distance is much larger than the range of the interaction between the atoms. The latter is effectively measured by a quantity called the *scattering length* of the potential and denoted by a, so the condition for a dilute gas can be written

$$\rho a^3 \ll 1. \tag{4}$$

Simultaneous fulfilment of (1) and (4) meant that extremely low temperatures (in the nanokelvin range) were required. Although interactions can not be ignored even in dilute gases the clear signals of BEC obtained in the experiments were recognized as the first direct confirmation of Einstein's prediction. For this feat the Nobel prizes for physics in 2001 were awarded to Ketterle, Cornell and Wieman.

The creation of BEC in ultra-cold, dilute Bose gases has lead to an enormous interest in such systems worldwide and to a deepening of the experimental and theoretical understanding of them. As two representative highlights I mention here only the generation of BEC in a gas of weakly bound molecules composed of fermions by R. Grimm and his group in Innsbruck in 2003 and the verification in the group of T. E. Hänsch in Munich in 2002 of a reversible quantum phase transition to BEC and back in optical lattices predicted 1998 by P. Zoller and co-workers in Innsbruck.

On the mathematical physics side some

problems that had remained unsolved for decades have been successfully attacked. A basic quantity for understanding the effects of interactions is the ground state energy of the Hamiltonian (2). Heuristic arguments that can be dated as back to a paper by W. Lenz from 1927, suggest that for dilute gases the interaction potential v enters essentially only through the scattering length a and the ground state energy per particle in the thermodynamic limit is given by the formula

$$e_0 = (2\pi\hbar^2/m)\rho a(1 + o(\rho a^3)). \qquad (5)$$

An upper bound for the energy in accord with (5) was given by F.J. Dyson in 1957, together with a lower bound that was, however, off the mark by a factor of about 14. A lower bound matching the upper bound was only proved 40 years later by E.H. Lieb and J. Yngvason. It is remarkable that the formula (5) holds both for 'soft' interactions where perturbation theory can be applied and the energy is mostly potential energy, as well as in the very different situation of interactions with a hard core where there is no perturbation theory and the energy is mostly kinetic.

The rigorous establishment of the energy formula (5), including the technique employed for its proof, was the seed for many further developments that are described in some detail in the book [1]. Particularly noteworthy among these is the proof, by Lieb and R. Seiringer in 2002, of BEC for an interacting system in a limit that is not quite the thermodynamic limit but nevertheless very relevant for current experiments. In fact, experiments with ultra-cold gases are carried out in magnetic or optical traps of finite extension L (≈ 1 mm, say) with a typical scattering length $a \approx 10^{-6} - 10^{-5}$ mm and particle number N of the order of a few millions. A thermodynamic limit may not be appropriate for describing such situations in small traps. A natural parameter for estimating the effect of the interaction is

$$g = Na/L \qquad (6)$$

and the limit when $N \to \infty$ keeping g fixed is called the *Gross-Pitaevskii (GP) limit*, after E.P. Gross and L.P. Pitaevskii who in 1961 independently put forward an equation where this parameter enters. (See Eq. (7) below.) Formally this limit can can be reached by taking either $L \sim N$ at fixed a, or $a \sim N^{-1}$ at fixed L. The significance of g can be understood by noting that it is the ratio of the interaction energy per particle, that is $\sim Na/L^3$ according to (5), and the lowest excitation energy $\sim 1/L^2$ for a free particle in the trap. Since $\rho a^3 = N^{-2}g^3$ it is also clear that (4) is fulfilled in the GP limit. The GP limit is appropriate for describing a dilute gas in a trap when the interactions are weak but still comparable to the excitation energies of noninteracting particles in the trap.

The GP description is not limited to a gas in a rectangular box, but applies quite generally to Hamiltonians of the form (2) with an arbitrary confining external potential V. In this case the gas cloud in the trap is in general inhomogeneous. The condensate wave function φ_0 is the solution of a nonlinear Schrödinger equation, the *Gross-Pitaevskii equation*

$$\left(-\Delta + V + 8\pi Na|\varphi_0|^2\right)\varphi_0 = \mu\varphi_0. \qquad (7)$$

There is a natural length scale L associated with the potential V and a rescaling of the position variables replaces Na in (7) by the dimensionless GP parameter $g = Na/L$. Hence this parameter appears also naturally as the coupling constant in front of the nonlinear term in the GP equation. The density of the condensate is given by $\rho_0(\vec{x}) = N|\varphi_0(\vec{x})|^2$, while its momentum distribution is $\rho_{0,\mathrm{mom}}(\vec{p}) = N|\tilde{\varphi}_0(\vec{p})|^2$, where $\tilde{\varphi}_0$ is the Fourier transform of φ_0. The signal of BEC in the experiments of Ketterle, Cornell and Wienand was precisely this momentum distribution that was measured by turning off the trap potential and letting the gas cloud expand freely. It emerged as a sharp peak out of the thermal Maxwell-Boltzmann distribution when the temperatures were low enough so that Einstein's condition (1) was fulfilled.

The GP equation has a time dependent variant that is used to describe the time evolution of the condensate if the initial trap potential is changed so that the condensate is no longer stationary. There are many interesting phenomena associated with time dependent condensates, e.g. macroscopic quantum interferences between the wave functions of two condensates when a potential barrier initially separating them is removed. Important steps towards a mathematical derivation of the time dependent GP equation from the many body dynamics have recently been taken by L. Erdős, B. Schlein und H.-T. Yau, but a complete solution of this problem has not yet been achieved.

It was already mentioned that there is still no rigorous proof of BEC in the thermodynamic limit at a fixed positive density for Hamiltonians of the form (2) in the presence of interactions, although a proof exists for the low density GP limit. One of the reasons a proof of BEC is so difficult is that BEC is accompanied by *spontaneous breaking of a continuous symmetry*. The meaning of this term will not be described precisely here, but there is an analogy with a ferromagnet where the magnetization has a fixed direction in space although the responsible interaction is invariant under rotations. In other words: The state breaks the symmetry of the Hamiltonian. In the case of BEC the symmetry is more abstract (it is a gauge symmetry associated with particle number conservation) but the basic idea is the same.

Proving the existence of phase transitions for systems with a continuous symmetry is notoriously difficult and there are even general theorems due to P. Hohenberg, D. Mermin and H. Wagner that exclude their occurrence in one and two dimensions. This can be explained by fluctuations of long wavelength that cost little energy in low dimensions but restore the symmetry. In 1976 a technique for proving phase transitions accompanied by continuous symmetry breaking was invented by J. Fröhlich, B. Simon and T. Spencer. It is based on an ingenious use of a property that had been discovered by K. Osterwalder and R. Schrader in the context of relativistic quantum field theory a few years before and is called *Reflection Positivity*. The original work of Fröhlich, Simon and Spencer was concerned with classical models of magnets, but it was generalized to quantum models two years later by Dyson, Lieb and Simon. Using this technique spontaneous symmetry breaking was proved for a class of models, including one that can be interpreted as a Bose gas of hard spheres. In this model, however, the particles do not move in a continuum but rather on a discrete lattice. Also, the technique allows only to prove BEC for a very special value of the density, namely when the number of particles is exactly half the number of lattice sites. Nevertheless, this model is not entirely academic because it can be approximately realized experimentally in lattices generated by beams of laser rays (optical lattices). Recently the model has been extended by M. Aizenman, E. Lieb, R. Seiringer, J.P. Solovej and J. Yngvason to allow for a variation of the strength of the optical potential, and the existence of a reversible transition between a BEC phase of delocalized particles and a "Mott insulator", where the particles are essentially confined to individual lattice sites, has been proved rigorously.

In this brief overview it has only been possible to mention a few of the problems posed to mathematical physics by the BEC phenomenon and a (very personal) selection of results obtained in recent years. Many important topics have been omitted, e.g. quantized vortices and superfluidity, where there is much experimental and theoretical activity and also some mathematical theorems. The importance of computer simulations that provide additional insights and even quantitative results (like the statement that liquid helium is 100% superfluid but only 9% Bose-Einstein condensed in the ground state) should also be emphasized. Altogether the physics of ultra-cold quantum gases and BEC has for the past 10 years been and still remains one of the most active research areas in physics as, e.g., witnessed by the monographs [2], [3] and thousands of papers and preprints listed on the web site [4].

References

[1] E.H. Lieb, R. Seiringer, J.P. Solovej and J. Yngvason, *The Mathematics of the Bose gas and its Condensation*, Birkhäuser, Basel (2005).

[2] C. Pethick, H. Smith, *Bose Einstein Condensation of Dilute Gases*, Cambridge University Press, 2001.

[3] L.P. Pitaevskii, S. Stringari, *Bose Einstein Condensation*, Oxford Science Publications, 2003.

[4] http://bec01.phy.georgiasouthern.edu/bec.html/bibliography.html

In December 2005, H. Grosse, E. Lieb, W. Thirring and P. Zoller organized a four-day workshop at the ESI on recent theoretical and experimental advances in the physics of ultra-cold atomic gases and of quantum information. The workshop brought together leading experts in the field, quite a number of whom came from Austria.

Recent Developments in Quantum Field Theory

Harald Grosse

Currently the best description of high energy physics data at accelerator energies is given by the so-called Standard Model of elementary particle physics. Some predictions of that model have been tested experimentally up to ten orders of magnitude. Nevertheless the calculations within this quantum field theory gauge model suffer from infrared problems, ultraviolet problems, the convergence problem of the perturbation theory as well as from the fact that gravity is treated classically and is not included as a quantum field.

During the last years several attempts have given rise to the hope of improving this unsatisfactory situation. One approach is via string theory, another one via noncommutative quantum field theory.

String theory is formulated in ten dimensions, candidates for the six added internal coordinates are Calabi-Yau manifolds, for which certain duality transformations led to very interesting new mathematical developments in algebraic geometry. Classification of states lead to twisted K-theory, and a particular field theory limit starting from string theory yields noncommutative quantum field theory models.

In noncommutative quantum field theory one starts from the algebra of smooth functions over a manifold, deforms the algebra and tries to keep additional structures like the differential calculus, replacing integration by a trace and sections of bundles by elements of finitely generated projective modules. The quantum field theories obtained this way suffer from what is called infrared-ultraviolet mixing: there are infrared singularities closely connected to the ultraviolet singularities. There are now successful techniques for dealing with this problem for matter fields. The method consists in adding one additional relevant/marginal operator and carrying out the renormalization procedure for the extended models. The resulting models show a particular duality which might be related to dualities in string theory.

In the presence of background flux the field theories coming from string theory are defined on noncommutative geometries and such geometries can be constructed by so-called bundle gerbes. This part of algebraic geometry is the main subject of three activities at ESI.

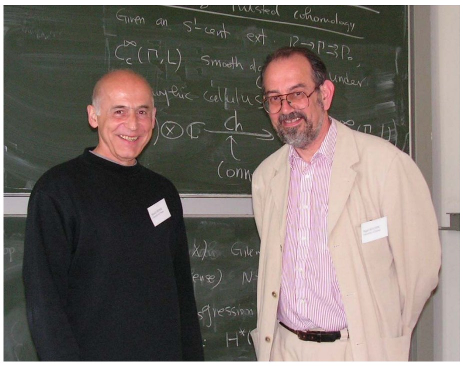
- A lecture course on *K-Theory and physics* by ESI Senior Research Fellow Mathai Varghese from March – June 2006;
- A programme on *Gerbes, Groupoids and Quantum Field Theory* in May, June and July 2006;
- A workshop in May 2006 on *Algebraic Geometry and Physics* in collaboration with Ugo Bruzzo from SISSA in Trieste.

MATHAI VARGHESE

Although the main application of these structures concerns string theory and therefore the unification of quantum field theory with classical general relativity, there is a second application which was extensively treated already in Mathai Varghese's lectures: a comparison of theory and experiments for the quantum Hall effect shows agreement of up to ten orders of magnitude.

The plateaus for the conductivity within the integer effect are of this precision. The explanation uses topological arguments, respectively noncommutative geometry techniques, K-theory and index theory. The fractional effect is less well understood. The Kubo cocycle was treated and the fractional charges were obtained as orbifold characteristics. Furthermore, connections with the Baum-Connes conjecture were explained.

The second part of these lectures deals with the T-dualities in string theory using noncommutative geometry. In the presence of H-flux space-time becomes noncommutative. These geometries are classified by the Dixmier-Douady class, and geometrically constructed via bundle gerbes. The D-brane charges and the Ramond-Ramond fields in Type II string theory are classified by the K-theory of this noncommutative geometries. Examples of such geometries arising from switching on background fluxes are also presented in the course.



HARALD GROSSE AND NIGEL HITCHIN

The Interaction of Mathematics and Physics at the Turn of the Twentieth Century

A Series of Lectures

Joachim Schwermer

The emergence of mathematical physics as an independent discipline at the end of the 19th century brought with it profound discussions of the foundations of both mathematics and physics as well as fruitful cooperation between these two fields. In this period far-reaching concepts of modern physics and new, fundamental mathematical structures were constructed. Mathematicians such as Henri Poincaré, David Hilbert und Hermann Minkowski contributed to this development. The reception of their work by physicists like Hendrik A. Lorentz or Albert Einstein, and the exchange of views on space and time, lie at the core of the revolutionary physical theories created at the beginning of the 20th century. There is need of closer examination of the interaction between mathematics and mathematical physics, its particular formulation in Vienna at the hands of Ludwig Boltzmann and others, and the reception of Einstein's theories of special and general relativity in Austria, for example among the theorists of the Vienna circle.

In the summer 2005, Senior Research Fellow Della Dumbaugh Fenster from the University of Richmond and Joachim Schwermer (ESI) drew attention to this topic area by organising a series of lectures at the ESI. The talks found broad interest among researchers and students and initiated a new awareness of the historical context that goes along with the sciences in question.

In February 2005, Leo Corry, Director of the Cohn Institute for History and Philosophy of Science at Tel-Aviv University, talked on *David Hilbert and the Axiomatization of Physics*. Among the contributions to physics Hilbert made, one finds his solution of the Boltzmann equation and his involvement with the formulation of the field equations of general relativity. It is also known that the sixth of Hilbert's famous 1900 list of problems was a call for the 'axiomatization of physics'. Still, until relatively recently, one considered all these contributions as no more than sporadic incursions into a totally foreign field. Based on recent historical research, Corry brought to light a very different picture whereby physics appears as a fundamental pillar of Hilbert's scientific world view and as an organic component of his research and teaching activities at Göttingen throughout his career. In particular, the axiomatization of physics appears as the connecting thread among all of his activities in this field, as well as the link to much of his work in pure mathematics.

Drawing on a wide range of archival material, including the correspondence between the young American algebraist Adrian A. Albert and the German number theorist Helmut Hasse, Della D. Fenster discussed in her talk *A Delicate Collaboration: Adrian Albert and Helmut Hasse and the Principal Theorem in Division Algebras in the Early 1930's* the emergence of the theory of normal simple algebras over algebraic number fields and their final classification. Traditionally this result is associated with the celebrated German trio of mathematicians Richard Brauer, H. Hasse and Emmy Noether. Indeed, they formed one of the collaborative efforts that led to the proof of the principal theorem in linear algebras in the 1930's. This talk, however, highlighted the other joint work linked with the proof of this result, namely that of Albert and Hasse.

Henri Poincaré was widely regarded as being one of the leading mathematicians of his day, as well as one of the leading theoretical physicists and a successful popular philosopher. The contributions he felt that he could make to theoretical physics drew upon his mastery of mathematics as well as his philosophy of science. In his talk *Poincaré's electro-magnetic Theory: Philosophy and Physics around 1900*, Jeremy Gray addressed these issues in detail. His discussion, in particular, shed light on the complicated question of the similarities and differences between Poincaré's ideas and Einstein's special theory of relativity.

After her stay in Vienna **Della D. Fenster**, ESI Senior Research Fellow in 2005 and co-organizer of this lecture series, wrote the following tribute to the atmosphere of the Senior Research Fellows Programme.

I would like to thank the ESI for a memorable and productive six-month stay as a Senior Research Fellow from January-June, 2005.

Thank you for creating a physical setting at the ESI that encourages the reflective atmosphere so essential to academic scholarship, particularly in mathematics. That the ESI occupies the top floor of an Abbey is a crucial factor in creating an ideal Institute for Mathematical Physics. Some of the most successful Mathematics Institutes in the World, such as the Mathematisches Forschungsinstitut in Oberwolfach, Germany and the Mathematical Sciences Research Institute in Berkeley, California, are located in isolated areas where stillness is the order of the day. It is simply remarkable that the ESI can create a similar type of space for its visitors in the heart of Vienna. And it is essential that the ESI is located in a rich and vibrant city like Vienna. When a scholar is at an impasse in their research, a creative piece of art at one of the museums or palaces will refine and/or redirect their focus in a precise and helpful way. The reflective spirit of the ESI is complemented by the art and culture of Vienna in a way not typically found at mathematics institutes.

While a Senior Research Fellow, I worked with Professor Joachim Schwermer on our second paper, "Beyond Class Field Theory: Helmut Hasse's Arithmetic in the Theory of Algebras in early 1931." The Senior Research Fellowship at the ESI made this important research project a reality. The nearly fifty-page paper that resulted from this work required learning the subtle details of class field theory in the 1930's. In particular, we devoted especial attention to how and why mathematicians used the theory of hypercomplex number systems to address outstanding questions in non-abelian class field theory—and beyond. This work hinged on handwritten archival materials written primarily in German. Translating these works in a German-speaking setting not only allowed for a more thorough understanding of the material on my part but also improved the overall insights necessary for a paper of this calibre.

Professor Schwermer and I also began work on our third collaborative paper, "Composition of Forms over Rings." This work grew out of initial discussions in Oberwolfach, 2001. At that time, we both attended a conference celebrating the 200th anniversary of Gauss's Disquistiones Arithmeticae. This paper highlights Gauss's work on the composition of forms, as presented by A. Hurwitz in his personal (mathematical) diaries and taken up (much later) by Martin Kneser in his work on Clifford algebras.

Professor Schwermer and I also hosted a Lecture Series on "Mathematics and Physics in the 19th Century." This brought the distinguished scholars Leo Corry and Jeremy Gray to Vienna for talks on various aspects of the work of David Hilbert and Henri Poincaré. These talks were not only well attended, but they also served as an opportunity to bring physicists, mathematicians, and historians together in a single audience. In recent years, the Schrödinger Institute has played a leading role in promoting the history of mathematics and physics. In particular, the Schrödinger audience often includes young scholars who will take a broad, positive view of the history of mathematics with them throughout their careers. Through the efforts of the Schrödinger Institute, these young scholars have seen that the history of mathematics is not a retired mathematician recounting the advancements in his/her research area, but, rather a vibrant, scholarly field that pursues technical questions within a broad framework that addresses issues related to biography, institutional settings, and political dynamics among others.

Outside of these collaborative efforts, I made significant progress on a book-length biography of Leonard Dickson. Dickson was an influential American mathematician in the early twentieth-century. This biography serves as the culmination of a decade of research on Dickson and his work in algebra and number theory. The Senior Research Fellowship allowed uninterrupted time to pursue this work in an inspiring setting. In particular, while at the Schrödinger, I worked on the chapters of the book that make use of the Carnegie Institution Archives, the largest existent collection of letters in Dickson's hand. Since Dickson burned his papers when he retired, these letters form the core of the primary sources available for this biography. I would like to add that discussions with Jeremy Gray during his visit to the Schrödinger contributed to the advancement of this biography. He provided invaluable insight into the issue related to the lack of primary sources. A single sentence summarizes what I took away from these discussions: "Don't focus on the Dickson you don't have. Focus on the Dickson you do have."

The six months in Vienna provided ample time to get to know the city. This, combined with my research in biography, allowed me to introduce a new course in the University of Richmond curriculum that will be offered in the Spring of 2007. This course, "What moves us? A Biographical Excursion in the City of Vienna" recently received a large grant to bring 10 students and one faculty member to Vienna for a week for further study. While this is not a direct mathematical result, it is a direct result of an opportunity provided by the ESI.

Finally, I must credit the ESI for their willingness to bring a scholar with three children to Vienna for six months. Joachim Schwermer and Isabella Miedl, in particular, took on extra work to settle my three children in schools and to help coach me through ordinary life in Vienna. It is a credit to the ESI that they would take on this extra challenge.

Thank you very much for this opportunity.

Della Fenster

The lecture series 'Interaction of Mathematics and Physics at the Turn of the Twentieth Century' will be continued in 2006

Report on the 'RDSES/ESI Educational Workshop on Discrete Probability'

Elmar Teufl

This workshop took place at the ESI from March 12–25, 2006. It was organised by Vadim A. Kaimanovich (Bremen), Klaus Schmidt (University of Vienna and ESI) and Wolfgang Woess (Technical University Graz), and aimed to attract young people coming from the wide area of discrete probability. A total of 53 participants from all over the world must be seen as a proof of success. In the wonderful ambience of the ESI the participants had the possibility for fruitful collaboration, discussion and research.

The workshop lasted for two weeks and the programme was divided into four mini-courses (two each week), extended talks from eleven renowned 'senior guests', and talks from the young attendees on their recent work. The mini-courses were given by Persi Diaconis (Stanford), Alex Gamburd (Princeton), Greg Lawler (Cornell), and Christophe Pittet (Marseille). The term 'educational' in the title (and the positive influence of the organisers) motivated all speakers to arrange their talks in a well comprehensible style, which resulted in a remarkably large and interested audience. Additionally, all lectures addressed active fields of research, so that the attendance was always of great benefit.

The topic of the workshop was discrete probability with emphasis on random walks and related fields of probability with a structure-theoretical component. It was intended as a follow-up of the ESI special semester '2001 – Random Walks'. The educational character of the programme should be mentioned positively: especially younger researchers could easily get in contact with new fields of mathematics and start new collaborations.

The event was well organised. In this context it deserves to mention the friendly secretaries of the ESI, who answered all questions patiently and helped with words and deeds. Furthermore, the facilities of the institute (computers, blackboards, etc.) are very comfortable.

PERSI DIACONIS

In the course of the workshop there was a beautiful public lecture of Persi Diaconis on 'Mathematics and Magic Tricks' at math.space, which attracted much public interest.

All in all, I think these two events (the ESI special semester '2001 – Random Walks' and the 'RDSES/ESI Educational Workshop on Discrete Probability') should be continued in future at the Erwin Schrödinger Institute.

The RDSES/ESI Educational Workshop on Discrete Probability was funded jointly by the ESF activity 'Phase Transitions and Fluctuation Phenomena for Random Dynamics in Spatially Extended Systems' and the ESI Junior Research Fellows Programme.

The ESI Senior Research Fellows Programme

Over the past three years the ESI has intensified its cooperation with the local scientific community and, in particular, with the graduate programmes of the University and the Technical University of Vienna through establishing a *Senior Research Fellows Programme* which offers — among other things — advanced lecture courses for graduate and postgraduate students. For the year 2006/07 the following lecture courses are planned.

Autumn 2006

Ioan Badulescu (Université de Poitiers): *Jacquet Langlands correspondence and unitary representations.*

Thomas Mohaupt (University of Liverpool): *Black Hole Entropy, Geometry and String Theory* (this lecture course will continue in May/June 2007).

Spring 2007

Vadim Kaimanovich (International University Bremen): *Amenability and Random Walks.*

The lecture note volume Kähler Geometry *by Werner Ballmann, Senior Research Fellow at ESI in 2004/05, will be published by European Mathematical Society Publishing House in August. It is the second volume of the ESI lecture note series* Lectures in Mathematics and Physics.

The ESI Junior Research Fellows Programme

The *Junior Research Fellows Programme* is now in its third year of operation. Funded by the Austrian Ministry of Science, it provides support for PhD students and young post-docs to participate in the scientific activities of the Institute and to collaborate with its visitors and members of the local scientific community. Currently there are 14 Junior Research Fellows visiting the ESI as part of this program. Their visits range from two to six months, and their scientific interests include relations between algebraic geometry and physics, dynamical systems and ergodic theory, mathematics and finance, to name a few. **Adam Joyce**, a graduate student from Imperial College in London, describes in his article some of his impressions.

Adam Joyce

I am a little over halfway through a six month stay in Vienna, working at the ESI as a Junior Research Fellow, and what follows are a few thoughts on my time here so far. I am a number theorist and first became aware of the Junior Fellowship Programme as a result of an advertisement for a workshop on arithmetic and algebro-geometric aspects of automorphic forms, which took place at the ESI in January/February 2006. I was keen to take part in this and applied for a fellowship to enable me to attend. Almost as soon as the arithmetic geometry workshop ended, another, on diophantine geometry, began and continues to run. So one shouldn't be fooled by the name; despite being called an Institute for Mathematical Physics, the ESI has a diverse range of mathematical activities, and has certainly kept this particular number theorist stimulated and occupied with organised events — there have been number theoretic seminars and lecture courses running non-stop for over three months. All of these activities have been conducted in a relaxed atmosphere, with all those in attendance happy to talk to a young researcher like myself. The Institute's support for those of us just beginning our research careers is admirable.

The ESI, occupying the top floor of a priests' seminary, is a wonderfully tranquil place to sit and think — it is close to the centre of the city and easily accessible by public transport, yet feels peaceful and calm. The impressive original fittings of the building — the stone floors and light airy offices — are well complemented by smart, modern fittings, with a comfortable common room in which to relax. It is too easy to spend all of one's time at the Institute! Nothing gets in the way of doing mathematics. The staff at the ESI, both academic and administrative, have been unfailingly friendly. The mathematicians, though busy, have always involved me in the local events and found time for a mathematical chat. I was invited to and took part in several of the seminars taking place at the University of Vienna. The administrative staff have dealt with any problems or queries I've had swiftly and effectively. Everyone was very helpful in getting me settled in to life at the Institute and in Vienna itself.

One of the great benefits of a visit to the ESI is the chance it provides to spend some time in the city of Vienna. Coming from London, I have grown accustomed to being able to 'down tools' at the end of the day and immediately be able to do any of a wide range of cultural activities. And Vienna can certainly match London for this. Of course, I have been to Schönbrunn, seen 'The Kiss' and been to the opera — like all good tourists should — but I have been fortunate to be shown a few things not in the guide books. The friends I have made amongst the staff at the ESI and students of the University have been kind enough to show me some of their own favourite places in Vienna — be they tucked-away restaurants, coffee houses or nightclubs. I am not even sure the six months of my stay will be long enough to sample all of the astonishing array of coffee houses. Apart from sating my worrying new addiction to coffee and cake, I have found them a pleasant setting for doing mathematics (though not as pleasant as the ESI, of course!). Now, if I can only finish this tome of Musil whilst I am here . . .

(Most of the) ESI Junior Research Fellows, May 2006

ESI News

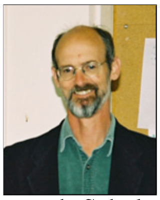
Starting from January 2006, the **Scientific Advisory Committee** of the ESI has the following composition:

Edward Frenkel (Berkeley)
Harald Grosse (Vienna)
Giovanni Gallavotti (Roma)
Nigel Hitchin (Oxford)
Gerhard Huisken (Potsdam)
Antti Kupiainen (Helsinki)
Elliott Lieb (Princeton)
Michael Struwe (ETH Zürich)

At the Advisory Committee Meeting on April 1, 2006, the following programmes were accepted for 2008:

- *Metastability and rare events in complex systems* (Bolhuis, Dellago, van der Eijnden)
- *Combinatorics and statistical physics* (Bousquet-Mélou, Drmota, Krattenthaler, Nienhuis)
- *Hyperbolic dynamical systems* (Posch, Szász, Lai-Sang Young)
- *Operator algebras and conformal quantum field theory* (Kawahigashi, Longo, Rehren, Yngvason)

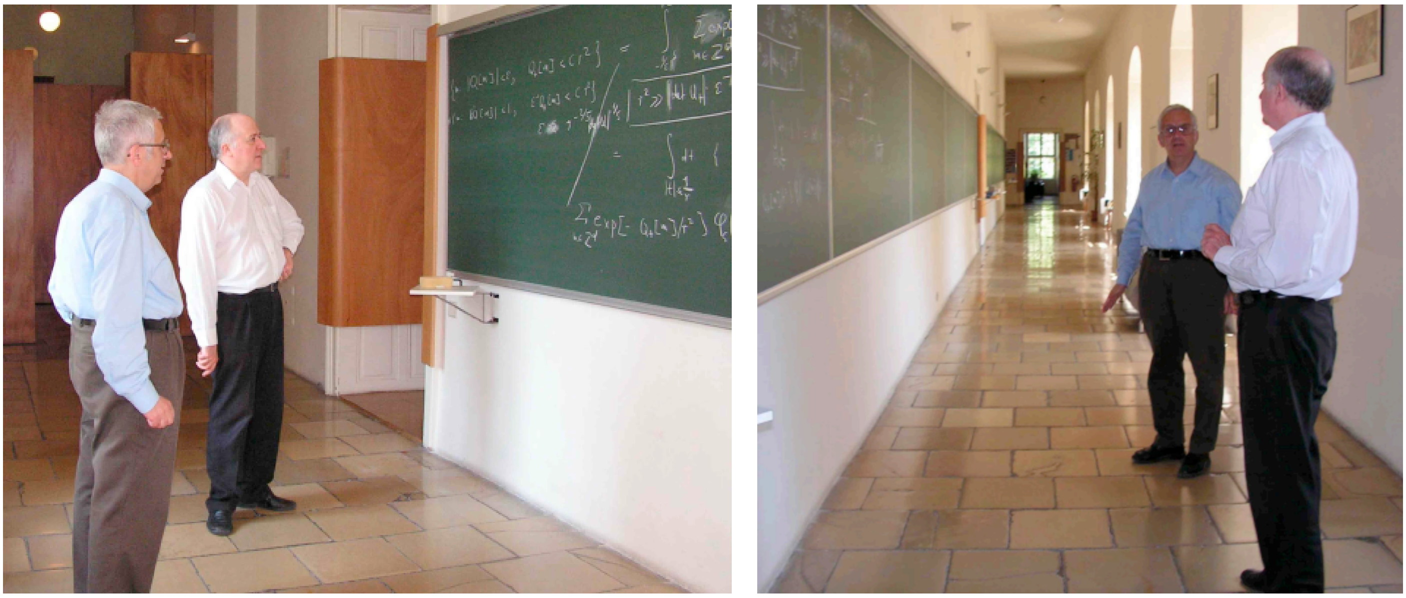
The next deadline for applications for **ESI Junior Research Fellowships** will be October 31, 2006. A call for application will be posted on the ESI web site in September 2006.



Alan Carey (Mathematical Sciences Institute, Australian National University, Canberra) is appointed as a Clay Mathematics Institute Research Scholar at ESI for the period May – July 2006. He takes part in the research programme *Gerbes, Groupoids and Quantum Field Theory* organized by P. Aschieri, H. Grosse, B. Jurco, J. Mickelsson and P. Xu.

Since the beginning of 2006 the editorial office of the journal *Reviews in Mathematical Physics* (RMP) is at ESI with **Jakob Yngvason** as Editor-in-Chief. RMP, published by World Scientific with H. Araki as founding editor, is one of the worlds leading journals in mathematical physics.

Jakob Yngvason has been elected *Foreign Member of the Royal Danish Academy of Sciences and Letters*. He has furthermore been appointed Member of the Steering Group of the newly founded *Niels Bohr International Academy, Copenhagen* and of the *Steering Council of the Doppler Institute of Mathematical Physics and Applied Mathematics, Prague*.



FRIEDRICH GÖTZE AND GREGORY MARGULIS AT WORK: WILL THE BLACKBOARDS BE BIG ENOUGH?

Current and future activities of the ESI

Thematic Programmes

Gerbes, Groupoids, and Quantum Field Theory: May 8 – July 31, 2006,

Organizers: P. Aschieri, H. Grosse, P. Xu, B. Jurco, J. Mickelsson

Workshops:

Algebraic Geometry and Physics: May 9 – May 13, 2006
Generalized Geometries: July 10 – July 14, 2006

Complex Quantum and Classical Systems and Effective Equations: May 15 – August 15, 2006

Organizers: E. Carlen, L. Erdős, M. Loss

Workshops:
June 1 – June 6, 2006, and July 24 – July 31, 2006

Homological Mirror Symmetry: June 12 – June 22, 2006

Organizers: A. Kapustin, M. Kreuzer, A. Polishuk, K. Schlesinger

---

Global Optimization – Integrating Convexity, Optimization, Logic Programming, and Computational Algebraic Geometry: October 1 – December 23, 2006

Organizers: A. Neumaier, F. Benhamou, I. Bomze, I. Emiris, C. Floudas, L. Wolsey

Workshop:
December 4 – December 8, 2006

Other Scientific Activities

Boltzmann's Legacy – A Symposium in Commemoration of L. Boltzmann's Death on September 5, 1906: June 7 – June 9, 2006

Organizers: G. Gallavotti, A. Kupiainen, W.L. Reiter, K. Schmidt, J. Schwermer, J. Yngvason

Workshop: Complex Analysis, Operator Theory and Applications to Mathematical Physics:
November 6 – November 17, 2006

Organizers: F. Haslinger, E. Straube, H. Upmeier

Workshop: Modern Methods of Time-Frequency Analysis

Organizers: H. Feichtinger, K. Groechenig, J. Benedetto

Workshop: Causes of Ecological and Genetic Diversity: December 10 – December 17, 2006

Organizers: R. Bürger and U. Dieckmann

Workshop: Langlands Duality and Physics: January 9 – January 20, 2007

Organizers: E. Frenkel, N. Hitchin, N. Nekrasov, J. Schwermer, K. Vilonen

---

Workshop: Automorphic Forms, Geometry and Arithmetic: February 11 – February 24, 2007

Organizers: S. Kudla, M. Rapoport, J. Schwermer

Editors: Klaus Schmidt, Joachim Schwermer, Jakob Yngvason

(Many of the) **Pictures**: Isabella Miedl

Contributors
Harald Grosse: harald.grosse@univie.ac.at
Wolfgang L. Reiter: wolfgang.reiter@univie.ac.at
Elmar Teufl: teufl@math.uni-bielefeld.de

ESI Contact List:
Administration
Isabella Miedl: isabella@esi.ac.at
Scientific Directors
Joachim Schwermer: joachim.schwermer@univie.ac.at
Jakob Yngvason: jakob.yngvason@univie.ac.at
President
Klaus Schmidt: klaus.schmidt@esi.ac.at

The ESI is supported by the Austrian Federal Ministry for Education, Science and Culture (bm:bwk). The ESI Senior Research Fellows Programme is additionally supported by the University of Vienna and the Technical University of Vienna.

This newsletter is available on the web at: ftp://ftp.esi.ac.at/pub/ESI-News/ESI-News1.1.pdf

IMPRESSUM: **Herausgeber, Eigentümer und Verleger**: INTERNATIONALES ERWIN SCHRÖDINGER INSTITUT FÜR MATHEMATISCHE PHYSIK, Boltzmanngasse 9/2, A-1090 Wien.
Redaktion und Sekretariat: Telefon: +43-1-4277-28282, Fax: +43-1-4277-28299, Email: secr@esi.ac.at
Zweck der Publikation: Information der Mitglieder des Vereins Erwin Schrödinger Institut und der Öffentlichkeit in wissenschaftlichen und organisatorischen Belangen. Förderung der Kenntnisse über die mathematischen Wissenschaften und deren kultureller und gesellschaftlicher Relevanz.